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Effect of Rotating Speed and Stiffeners on Natural Frequency of Composite Shell

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The effect of rotating speed and stiffeners on vibrations for composite rotors is investigated using Sander's shell theory. The frequency equation is derived implementing the Rayleigh-Ritz procedure based on energy method. The effects of initial hoop tension, centrifugal and Coriolis forces due to the rotation are considered to derive governing equation. The displacement functions satisfying the both ends simply supported boundary conditions are assumed to be trigonometric expressions. By using simple shell theory like as Sander's shell theory the amount of equations and time expenditure are considerably reduced and provides feasible analysis, solution and design especially for composite materials optimization. UD composite materials are used for stiffeners. The effects of these stiffeners are evaluated by an averaging method. Some of stiffeners shapes are considered to optimize the ratio of natural frequency to weight.

Keywords: vibration, composite, rotor, Sander's theory, stiffener shape

Introduction

Rotating shells and shafts are used in many industrial applications and they are the main parts of many machines, such as gas turbines, locomotive engines, electric motors, rotor systems and fuel tanks. In many cases, a rotating shell may be one of the main sources of vibration and noise. In order to reduce the vibration, noise and increasing strength and also to enhance the stiffness of the shell, many shells and shafts are usually made of laminated composite materials or it is reinforced by stiffener. The stiffened cylindrical shell with beam type elements is extensively used in mechanical structures, such as aircraft fuselages, commercial vehicles, road tankers, missiles, and submarines, etc.

It is, therefore, very important for engineers to understand the vibration of composite shells in order to design suitable shells with low vibration and noise production characteristics. Hence, vibration characteristics of rotating stiffened cylindrical shells are of great importance.

In recent years, as composite materials have advantages of high strength-to-weight ratio, advanced composite materials have been used widely in many fields of engineering. There are many studies on these rotating structures without stiffeners. Previously published papers, however, were primarily concerned with the stiffened isotropic shell and many studies have been carried out on laminated cylindrical shells with or without the rotation. But papers about optimization of related parameters were rare.

Numerous methods have been developed and used to study the vibrational behavior of thin shells. These methods range from energy methods based on the Rayleigh-Ritz procedure to analytical methods in which, respectively, closed-form solutions of the governing equations and iterative solution approaches were used. On the other hand, the wave propagation in cylindrical shells has also been investigated by many researches.

The study of the vibrations for the composite cylindrical shells has been reported by many researchers [1-2]. They have studied the effects of various parameters such as boundary conditions, aspect ratios, fiber orientation angles and material properties of the composite shells on the vibration characteristics. But only several researchers investigated the vibrations of the combined shell with an interior plate.

ESDU [3] (Engineering Science Data Unit) has published a computer program for the orthogonally stiffened shells. Irie et al. [4] studied the free vibration of non-circular cylindrical shells with longitudinal interior partitions by using the transfer matrix.

Mustafa and Ali [5] presented a concise yet comprehensive method for the determination of natural

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frequency of ring, stringer and orthogonally stiffened cylindrical shells based on the formulation of energy. Lam and Loy [6] presented the vibration characteristics for the GFRP composite laminate cylindrical shell using different shell theories.

Lee and Kim [7] studied rotating stiffened cylindrical shell by using the energy method. The stiffeners are assumed to be an integral part of the shell and have been directly included in analysis. Zhao et al. [8] analyzed vibration analysis of rotating cross-ply laminated circular cylindrical shells with stringer and ring stiffeners. Love's relations were used to obtain the governing equation and then, they solved it by two approaches including discrete elements and averaging methods. Reddy's layerwise theory is combined with a wave propagation approach by Ramezani and Ahmadian [9] to study all the conventional boundary conditions in our analysis by using Hamilton's principle. One of the major advantages of the layerwise theory is the possibility it provides for analyzing thick laminates and, also, inter-lamina stresses (in forced vibrations) with high accuracy. Qin et al. [10] observed grazing bifurcation in the response of rubbing rotor. Patel and Darpe [11] discussed use of spectrum cascade for identification of rub and showed existence of backward whirling components. Liew et al. [12] studied the dynamic stability of composite laminated, functionally graded and rotating cylindrical shells under periodic axial forces . Ahmad and Naeem [13] investigated the vibration characteristics of rotating FGM cylindrical shells for a number of boundary conditions by using wave propagation approach but unstiffened shells . Civalek O¨mer [14] proposes a discrete singular convolution method for the free vibration analysis of rotating conical shells. Frequency parameters of the forward modes are obtained for different geometric parameters. Wang et al. [15] derived the equation of motion for rotating circular cylindrical shells by using the Donnell's nonlinear shallow-shell theory, and it includes Coriolis force and large-amplitude shell motion effects. Isotropic material considered to cylindrical shells without stiffeners. Jafari et al. [16] investigated the Free vibration of rotating ring stiffened cylindrical shells with non-uniform stiffener distribution

In the present paper, the Sander's shell theory is used to perform an analytical solution for free vibration of the stiffened composite shells by using Ritz method. Although Sander's theory is simple but in design process, the time is so important and because its results have a good approximation, it has been used. The effects of initial hoop tension, centrifugal and Coriolis forces due to the rotation are considered to derive governing equation by using averaging method. A computer code is prepared to characterize natural frequency of isotropic and orthotropic stiffened shells versus variations of rotating speed.

Theoretical Formulations

The stiffened cylindrical shell, as shown in Figure 1, is considered to be thin, laminated and composed of an arbitrary number layers with parameters length *L*, radius *R*, thickness *h*, and is rotating about the *x*-axis at constant angular velocity Ω .A coordinate system (x,θ,z) is fixed on the middle surface of the shell. The displacements of the shell in the x,θ,z directions are denoted by *u, v,* and *w* respectively. The depths of the stringer and ring are denoted by d_s and d_r, respectively, and the corresponding widths by b_s and b_r, respectively. The displacements from the middle surface of the shell to the centroid of the stringer and ring are denoted by Z_s and Z_r, respectively.

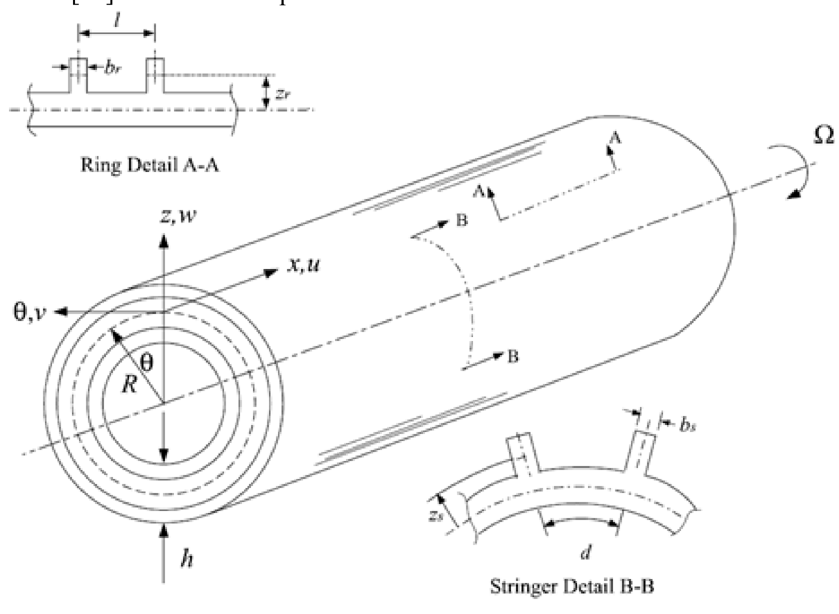

Figure 1. Coordinate system and stiffener and ring cross section area for the rotating and orthogonally stiffened *cylindrical shell*

The strain vector can be written as

$$\{\varepsilon\}^T = \{e_1 \quad e_2 \quad e_3 \quad k_1 \quad k_2 \quad k_3\} \tag{1}$$

Where the middle surface strains, e_1, e_2, e_3 and the middle surface curvatures k_1, k_2, k_3 are defined according to Sander's theory [17] as follows

$$\begin{aligned} e_1 &= \frac{\partial u}{\partial x} \\ e_2 &= \frac{1}{R}\left(w + \frac{\partial v}{\partial \theta}\right) \\ e_3 &= \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta} \\ k_1 &= -\frac{\partial^2 w}{\partial x^2} \\ k_2 &= \frac{1}{R}\left(\frac{\partial v}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2}\right) \\ k_3 &= \frac{1}{R}\left(\frac{\partial v}{\partial x} - 2\frac{\partial^2 w}{\partial x \partial \theta}\right) \end{aligned} \tag{2}$$

For stringers, the displacements in the x, θ, z directions are defined as

$$\begin{aligned} u_s &= u - z\frac{\partial w}{\partial x} \\ v_s &= v - \frac{z}{R}\frac{\partial w}{\partial \theta} \\ w_s &= w \end{aligned} \tag{3}$$

The strain of stringers in the axial direction is described as

$$\varepsilon_s = \frac{\partial u_s}{\partial x} \tag{4}$$

For rings, the displacements in the x, θ, z directions are defined as

$$\begin{aligned} u_r &= u - z_r\frac{\partial w}{\partial x} \\ v_r &= v - \frac{z_r}{R}\frac{\partial w}{\partial \theta} \\ w_r &= w \end{aligned} \tag{5}$$

The strain of rings in the radial direction is described as

$$\varepsilon_r = \frac{1}{R}\left(w_r + \frac{\partial v_r}{\partial \theta}\right) \tag{6}$$

$$[\tilde{Q}] = [T][Q][T]^{-1} \tag{11}$$

If the shell is assumed to be simply supported, the displacement components can be approximated in the term of time (t) as

$$\begin{aligned} u &= A\cos(\frac{m\pi x}{L})\cos(n\theta + \omega t) \\ v &= B\sin(\frac{m\pi x}{L})\sin(n\theta + \omega t) \\ w &= C\sin(\frac{m\pi x}{L})\cos(n\theta + \omega t) \end{aligned} \tag{7}$$

Where m represents the number of axial half wave, n represents the number of circumferential half wave and ω is the natural frequency of the rotating shell.

Strain Energy of Shell

The strain energy of the shell is expressed as

$$U_e = \frac{1}{2}\int_0^L\int_0^{2\pi}\{\varepsilon\}^T[S]\{\varepsilon\}Rd\theta dx \tag{8}$$

Where $[S]$ is the stiffness matrix and can be written as

$$[S] = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \tag{9}$$

Where the A_{ij}, B_{ij} and D_{ij} are defined as extensional, coupling and bending stiffness, respectively. For a shell composed of different layers of orthotropic material, these stiffnesses can be written as

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{N}\tilde{Q}_{ij}^{(k)}(h_k - h_{k+1}) \\ B_{ij} &= \frac{1}{2}\sum_{k=1}^{N}\tilde{Q}_{ij}^{(k)}(h_k^2 - h_{k+1}^2) \\ D_{ij} &= \frac{1}{3}\sum_{k=1}^{N}\tilde{Q}_{ij}^{(k)}(h_k^3 - h_{k+1}^3) \end{aligned} \tag{10}$$

Where h_k and h_{k+1} denote the distance from the shell reference surface (middle surface) to the outer and inner surfaces of the k-th layer. Then, N is the number of layers in the laminated shell and $\tilde{Q}_{ij}^{(k)}$ is the transformed reduced stiffness matrix for the k-th layer which is defined as

Where $[T]$ is the transformation matrix for the principal material coordinates and the shell coordinates system and is defined as

$$[T]=\begin{bmatrix}\cos^2\alpha & \sin^2\alpha & \sin 2\alpha\\ \sin^2\alpha & \cos^2\alpha & -\sin 2\alpha\\ -\frac{1}{2}\sin 2\alpha & \frac{1}{2}\sin 2\alpha & \cos 2\alpha\end{bmatrix} \quad (12)$$

Where α is the orientation of the fibers and $[Q]$ is the reduced stiffness matrix which is defined as

$$[Q]=\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix} \quad (13)$$

And the material constants in the reduced stiffness matrix are given as

$$Q_{11}=\frac{E_{11}}{1-\nu_{12}\nu_{21}}$$
$$Q_{12}=\frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}}$$
$$Q_{22}=\frac{E_{22}}{1-\nu_{12}\nu_{21}} \quad (14)$$
$$Q_{11}=G_{12}$$

Where E_{11} and E_{22} are the elastic modulus, G_{12} is the shear modulus and ν_{12} and ν_{21} are the Poisson's ratios. The strain energy of shell due to hoop tension is [16]

$$U_{h,e}=\frac{h}{2}\int_0^L\int_0^{2\pi}N_{\theta,e}\left\{\begin{aligned}&\left[\frac{1}{R}\frac{\partial u}{\partial\theta}\right]^2+\left[\frac{1}{R}\left(w+\frac{\partial v}{\partial\theta}\right)\right]^2\\&+\left[\frac{1}{R}\left(v-\frac{\partial w}{\partial\theta}\right)\right]^2\end{aligned}\right\}R\,d\theta dx \quad (15)$$

Where the initial hoop tension due to centrifugal force is defined as

$$N_{\theta,e}=\rho R^2\Omega^2 \quad (16)$$

Where ρ is the shell density. The kinetic energy of the rotating shell is given by

$$T_e=\frac{\rho h}{2}\int_0^L\int_0^{2\pi}\begin{bmatrix}\dot u^2+\dot v^2+\dot w^2+2\Omega(v\dot w-\dot v w)\\+\Omega^2(v^2+w^2)\end{bmatrix}R\,d\theta dx \quad (17)$$

Where $\dot u,\dot v,\dot w$ are the components of the velocity in the x,θ,z direction, respectively. The strain energy of the rings, by using averaging method is expressed as

$$\tilde U_r=\frac{1}{2l}\int_0^L\int_0^{2\pi}\left\{\int_{A_r}E_r\varepsilon_r^2dA_r+G_rJ_r\left(\frac{1}{R}\frac{\partial^2 w_r}{\partial x\partial\theta}\right)^2\right\}R\,d\theta dx \quad (18)$$

Where E_r, A_r and G_rJ_r are the elastic modulus, cross sectional area and torsional stiffness of rings, respectively. Then, l is the distance between rings. The strain energy of the rings due to hoop tension, by using averaging method is taken to be

$$\tilde U_{h,r}=\frac{1}{2l}\int_0^L\int_0^{2\pi}\int_{A_r}N_{\theta,r}\left\{\begin{aligned}&\left[\frac{1}{R}\frac{\partial u_r}{\partial\theta}\right]^2+\left[\frac{1}{R}\left(w_r+\frac{\partial v_r}{\partial\theta}\right)\right]^2\\&+\left[\frac{1}{R}\left(v_r-\frac{\partial w_r}{\partial\theta}\right)\right]^2\end{aligned}\right\}dA_rR\,d\theta dx \quad (19)$$

Where the initial hoop tension of rings due to centrifugal force is defined as

$$N_{\theta,r}=\rho_rR^2\Omega^2 \quad (20)$$

Where ρ_r is the density of rings.

The kinetic energy of rings, by using averaging method is given by

$$\tilde T_r=\frac{\rho_r}{2l}\int_0^L\int_0^{2\pi}\int_{A_r}\begin{bmatrix}\dot u_r^2+\dot v_r^2+\dot w_r^2+\\2\Omega(v_r\dot w_r-\dot v_rw_r)\\+\Omega^2(v_r^2+w_r^2)\end{bmatrix}dA_rR\,d\theta dx \quad (21)$$

The strain energy of the stringers, by using averaging method is expressed as

$$\tilde U_s=\frac{1}{2d}\int_0^L\int_0^{2\pi}\left\{\int_{A_s}E_s\varepsilon_s^2dA_s+G_sJ_s\left(\frac{1}{R}\frac{\partial^2 w_s}{\partial x\partial\theta}\right)^2\right\}R\,d\theta dx \quad (22)$$

Where E_s, A_s and G_sJ_s are the elastic modulus, cross sectional area and torsional stiffness of stringers, respectively. Then, d is the distance between stringers. Also, it should be mentioned that the strain energy of the stringers due to hoop tension is zero. The kinetic energy of stringers, by using averaging method is given by

$$\tilde T_s=\frac{\rho_s}{2d}\int_0^L\int_0^{2\pi}\int_{A_s}\begin{bmatrix}\dot u_s^2+\dot v_s^2+\dot w_s^2\\+2\Omega(v_s\dot w_s-\dot v_sw_s)\\+\Omega^2(v_s^2+w_s^2)\end{bmatrix}dA_sR\,d\theta dx \quad (23)$$

Where ρ_s is the density of the stringers. Total strain energy includes the energy functional of the shell, rings and stringers, by using averaging method can thus be written as

$$\tilde\Pi=T_e+\tilde T_r+\tilde T_s-U_e-U_{h,e}-\tilde U_r-\tilde U_{h,r}-\tilde U_s \quad (24)$$

Appling Hamilton's principles, by using averaging method results as

$$\delta\int_{t_0}^{t_0+2\pi/\omega}\tilde\Pi dt=0 \quad (25)$$

The following matrix relationship can be established as

$$\begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad (26)$$

For non-trivial solution of above equation, it should be

$$\begin{vmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} \end{vmatrix} = 0 \quad (27)$$

Expanding above equation, the characteristic frequency equation can be obtained as

$$\tilde{\beta}_6\omega_{mn}^6 + \tilde{\beta}_5\omega_{mn}^5 + \tilde{\beta}_4\omega_{mn}^4 + \tilde{\beta}_3\omega_{mn}^3 + \tilde{\beta}_2\omega_{mn}^2 + \tilde{\beta}_1\omega_{mn} + \tilde{\beta}_0 = 0 \quad (28)$$

It should be noted that for non-rotating shells, the coefficient for odd powers of ω_{mn} do not appear and the coefficient for the term ω_{mn}^5 will be zero for un-stiffened rotating shells.

The solution for equation (28) is, the frequencies split into two parts, one value corresponds to the backward wave and other to the forward wave of the rotating cylindrical shell respectively.

This split of the solution into two parts is perceived as a bifurcation phenomenon for the rotating cylindrical shell. When the shell is stationary, these two values are identical. However, when the cylindrical shell starts to rotate, the standing wave will be transformed into backward or forward waves depending upon the direction of rotating. In fact a numerical calculation shows that the absolute value of frequency of backward wave is always larger than that of forward wave. The figure (1) shows only the absolute values.

Results and Discussions

A code is written in MATLAB software to solve equations and to check the validity of the present analysis and the code results, the natural frequencies are compared with different works which are listed in Tables 1-4.

Table 1 include the natural frequencies of an isotropic cylindrical shell (non-rotating and un-stiffened), Table 2 includes non-dimensional natural frequencies of a rotating [0/90/0] laminated cylindrical shell, Table 3 includes the natural frequencies of an isotropic cylindrical shell (non-rotating) with 20 stringers and 13 rings and Table 4 includes the natural frequencies of a non-rotating [0/90/0] laminated cylindrical shell with stiffeners. All the results show an adoption of the present method to the other references.

Stiffeners effect on natural frequencies of $[90/0/\text{-}45/45]_s$ laminated rotating cylindrical shell is studied in Table 5 and Figure 1 for without rings, 5, 10, 20 and 30 rings. The results are drawn in Figure 1 for *m*=1 and *n*=3 with rotating speed and in Figure 2 for *m*=1 and *n*=1 until *n*=10 and 20 rings and different angular velocities (0-50 rev/sec).

The results show that by increasing of angular velocity until 20 (rev/sec), firstly forward natural frequencies are decreased and then increased. It should be mentioned that backward natural frequencies are continuously increased by increasing of angular velocity. By increasing the number of rings and angular velocity, natural frequencies are increased, too. For all content of *n*, the fundamental mode is constant and occurred in *m*=1 and *n*=3 but in higher amounts.

Table 1. Natural frequencies (Hz) of an isotropic cylindrical shell (non-rotating), h=0.02 (in.), L=11.74 (in.), R=5.836 (in.), density=0.000734 (lb^2/in^4), E=29500000 (lb/in^2), v=0.285

n	Natural frequencies (Hz)					
	m=1			m=2		
	Bert et al. [1] (Love's eq.)	Rath & Das [2] (SDST eq.)	Present (Sander's eq.)	Bert et al. [1] (Love's eq.)	Rath & Das [2] (SDST eq.)	Present (Sander's eq.)
1	3271.0	3270.53	3270.51	4837.9	4837.67	4837.37
2	1862.3	1861.95	1861.76	3725.5	3724.98	3724.41
3	1102.0	1101.75	1100.37	2743.7	2742.61	2740.93
4	705.9	706.66	699.27	2018.5	2018.02	2013.49
5	497.9	497.47	475.65	1515.4	1514.96	1503.51

Table 2. Non-dimensional natural frequencies ($\tilde{\omega} = \omega R\sqrt{\rho/E_{22}}$) of a rotating [0/90/0] laminated cylindrical shell, L/R=1, h/R=1; h=0.002 (m), E_{11}=19 (GPa), E_{22}=7.6 (GPa), G_{12}=4.1(GPa), v_{12}=0.26, density=1643 (kg/m^3)

Angular Velocity (rev/s)	n	Natural Frequencies (rad/s) and Non-dimensional Natural Frequencies, m=1							
		Lam & Loy [6]		Lee & Kim [7]		Zhao et al. [8]		Present	
		Backward	Forward	Backward	Forward	Backward	Forward	Backward	Forward
0.1	1	1.061429	1.061140	-	-	1.061428	1.061139	1.061429	1.061140
0.1	2	0.824214	0.803894	-	-	0.804212	0.803892	0.804214	0.803894
0.1	3	0.598476	0.598157	-	-	0.598472	0.598183	0.598476	0.598187
0.1	4	0.450270	0.450021	-	-	0.450263	0.450015	0.450270	0.450021
0.1	5	0.345363	0.345149	-	-	0.345355	0.345140	0.345363	0.345149
0.1	6	0.270852	0.270667	-	-	0.270840	0.270654	0.270851	0.270667
0.1	7	0.217651	0.217489	-	-	0.217635	0.217473	0.217651	0.217489
0.4	1	1.061862	1.060706	1.061850	1.060693	1.061862	1.060705	1.061862	1.060706
0.4	2	0.804696	0.803415	0.804691	0.803410	0.804694	0.803413	0.804696	0.803415
0.4	3	0.598915	0.597762	0.598912	0.597759	0.598911	0.597758	0.598915	0.597762
0.4	4	0.450662	0.449667	0.450658	0.449664	0.450654	0.449660	0.450661	0.449666
0.4	5	0.345724	0.344870	0.345719	0.344866	0.345714	0.344860	0.345723	0.344869
0.4	6	0.271207	0.270468	0.271200	0.270461	0.271193	0.270454	0.271205	0.270466
0.4	7	0.218029	0.217382	0.218020	0.217373	0.218011	0.217364	0.218026	0.217379
1.0	1	1.062728	1.059836	1.062716	1.059825	1.062728	1.059837	1.062729	1.059837
1.0	2	0.805667	0.802464	0.805660	0.802457	0.805664	0.802461	0.805666	0.802463
1.0	3	0.599820	0.596937	0.599814	0.596931	0.599813	0.596930	0.599817	0.596934
1.0	4	0.451513	0.449027	0.451506	0.449019	0.451502	0.449015	0.451508	0.449022
1.0	5	0.346593	0.344459	0.346583	0.344448	0.346577	0.344442	0.346586	0.344451
1.0	6	0.272197	0.270349	0.272182	0.270334	0.272174	0.270326	0.272186	0.270339
1.0	7	0.219269	0.217651	0.219248	0.217631	0.219240	0.217621	0.219255	0.217637

Table 3. Natural frequencies (Hz) of an isotropic cylindrical shell (non-rotating), h=0.00204 (m), R=0.203(m), L=0.813 (m), E=207 (GPa), v=0.3, density= 7430 (kg/m^3), with 20 Stringers: 0.004 (m) X 0.006 (m) and 13 Rings: 0.006 (m) X 0.008 (m)

n	Natural Frequencies (Hz), m=1			
	ESDU [3]	Mustafa & Ali [4]	Lee & Kim [7]	Present (Averaging)
1	938	942	947	1037.8
2	443	439	458	472.4
3	348	337	355	353.6
4	492	482	507	510.7
5	745	740	776	789.3

Table 4. Natural frequencies of a non-rotating [0/90/0] laminated cylindrical shell, L/R=4, h/R=0.005; h=0.001 (m), E_{11}=19 (GPa), E_{22}=7.6 (GPa), G_{12}=4.1(GPa), v_{12}=0.26, density=1643 (kg/m^3), b_r=b_s=0.002 (m), d_r=d_s=0.008 (m), E_r=E_s=3E_{11}, v=0.3

n	Natural Frequencies (Hz), m=1							
	Zhao et al. [8]		Present		Zhao et al. [8]		Present	
	Stringers/Rings: 4/4		Stringers/Rings: 4/4		Stringers/Rings: 10/5		Stringers/Rings: 10/5	
	Averaging		Averaging		Averaging		Averaging	
1	553.7		651.4		549.7		653.5	
2	288.4		292.3		299.8		298.1	
3	291.6		234.7		305.2		246.9	
4	470.6		259.7		486.7		382.0	
5	734.6		561.8		756.8		596.6	
6	1062.8		814.4		1093.8		864.4	
7	1451.5		1113.3		1493.0		1181.2	
8	1899.7		1457.7		1953.3		1546.2	
9	2406.9		1847.4		2474.1		1959.0	

Table 5. Natural frequencies of a non-rotating [90/0/-45/45]$_s$ laminated cylindrical shell, R=0.2 (m), L=1 (m); h=0.002 (m), E_{11}=139.4 (GPa), E_{22}=8.35 (GPa), G_{12}=3.1(GPa), density=1542 (kg/m^3), b_r=0.002 (m), d_r=0.012 (m)

n	Natural frequencies (Hz), m=1				
	Without rings	5 rings	10 rings	20 rings	30 rings
1	898.5	905.9	913.3	927.9	942.2
2	369.6	402.1	432.1	486.7	535.7
3	226.4	290.7	343.1	429.1	500.5
4	276.3	344.8	401.6	496.2	575.3
5	417.0	478.1	532.2	626.2	708.2
6	604.5	660.7	712.5	805.9	889.6
7	829.6	883.8	934.9	1029.4	1115.7
8	1090.3	1144.7	1196.6	1294.0	1389.5
9	1386.1	1442.2	1496.1	1598.3	1694.3
10	1716.9	1775.7	1832.6	1941.3	2044.1

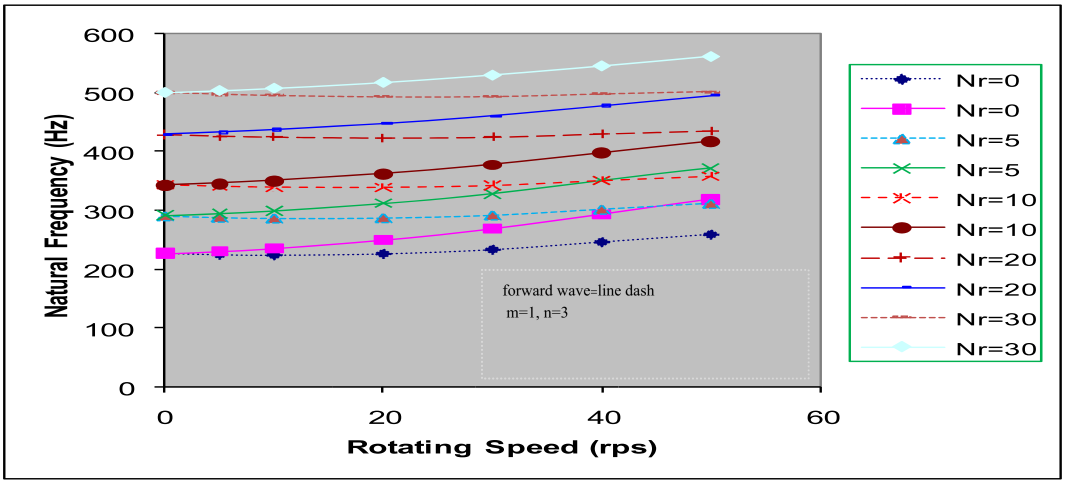

Figure 1. Natural frequencies of a rotating [90/0/-45/45]$_s$ laminated cylindrical shell, R=0.2 (m), L=1 (m); h=0.002 (m), E_{11}=139.4 (GPa), E_{22}=8.35 (GPa), G_{12}=3.1(GPa), density=1542 (kg/m^3), b_r=0.002 (m), d_r=0.012 (m)

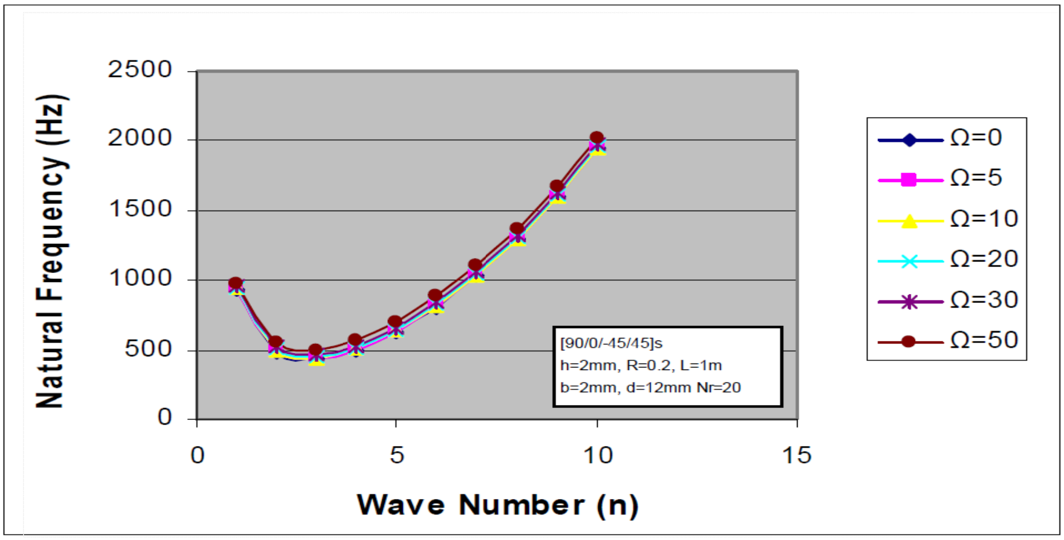

Figure 2. Natural frequencies of a rotating [90/0/-45/45]$_s$ laminated cylindrical shell, R=0.2 (m), L=1 (m); h=0.002 (m), E_{11}=139.4 (GPa), E_{22}=8.35 (GPa), G_{12}=3.1(GPa), density=1542 (kg/m^3), b_r=0.002 (m), d_r=0.012 (m)

Optimization of stiffener shape

Four shapes (Figure 3) are considered for stiffeners (both rings and stringers) in a rotating shell including rectangular shape , C-shape, I-shape and Ω -shape. The details of these shapes are listed in Table 6. The dimensions of shell include thickness, radius and length are considered as 0.002 (m), 0.2 (m) and 1.0

(m), respectively. And also the angular velocity is constant as 0.1(rev/s).

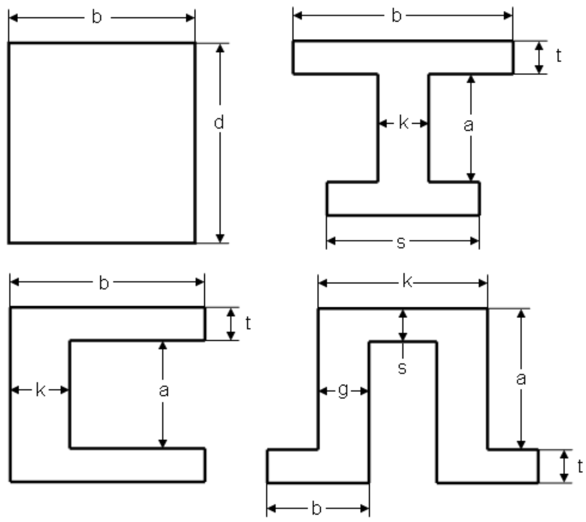

Figure 3. Different shapes of stiffeners and their defined dimensions [18]

Table 6. Geometric cross sectional data

Shape Types	Constants	Dimension	Contents
Rectangular	$b_r=b_s$	m	0.002
	$d_r=d_s$	m	0.010
	$A_r=A_s$	m^2	$2.000 * 10^{-5}$
C-Shape	$b_r=b_s$	m	0.004
	$t_r=t_s$	m	0.001
	$k_r=k_s$	m	0.004
	$a_r=a_s$	m	0.003
	$A_s=A_r$	m^2	$2.000 * 10^{-5}$
I-Shape	$b_r=b_s$	m	0.008
	$t_r=t_s$	m	0.001
	$k_r=k_s$	m	0.002
	$a_r=a_s$	m	0.004
	$s_r=s_s$	m	0.004
	$A_r=A_s$	m^2	$2.000 *10^{-5}$
Ω -Shape	$b_r=b_s$	m	0.004
	$t_r=t_s$	m	0.001
	$k_r=k_s$	m	0.004
	$a_r=a_s$	m	0.004
	$s_r=s_s$	m	0.001
	$g_r=g_s$	m	0.001
	$A_r=A_s$	m^2	$2.000 * 10^{-5}$

The results are listed in Table 7 for stiffened shell by10 rings and 10 stringers. The target of this optimization is to maximize the defined parameter, $f_{m,n}$ (ratio of natural frequencies to weights) . As it can be seen in the Table 7, for different contents of *m* and *n*, the C-shape has the highest value of $f_{m,1}$ and the I- shape has the lowest value. But for contents of $m=1$ and $n = 3$, the rectangular -shape has the highest value of $f_{m,3}$ and the C -shape has the lowest value.

Table 7. The ratio of natural frequencies to weights for different stiffener shapes

Stiffener Types	Shape Types	Ratio of natural frequencies to weights (Hz/Kg), $f_{m,n}$ (m=1)			
		n=1	n=2	n=3	n=4
Rings	Rectangular	34.873	15.775	15. 442	26.249
Stringers	Rectangular				
Rings	C-Shape	34.893	14.773	11.291	16.855
Stringers	C-Shape				
Rings	I-Shape	32.518	15.223	14.610	25.721
Stringers	I-Shape				
Rings	Ω -Shape	34.492	14.693	11.626	17.681
Stringers	Ω -Shape				

Conclusion

Free vibration analysis of simply supported rotating cross-ply laminated stiffened cylindrical shell is performed by using an energy approach, Reilly-Ritz method and Sander's relations. A good adoption is observed between the present results and other literatures in different type of results including isotropic shells, rotating laminated shells, stiffened isotropic shells and stiffened laminated shells.

Then, the results show that

- The stiffeners have a little effect on natural frequencies until the fundamental frequency but more than fundamental frequency, the change in natural frequencies is performed in a higher rate.
- By increasing the numbers of stiffeners, by using averaging method, the results are more accurate.
- By using an analytical approach based on simple functions for shape mode, it can be obtained accurate results without using complex relations.
- By increasing the numbers of rings, natural frequencies are increased, too.
- Backward natural frequencies are continuously increased by increasing of angular velocities in un-stiffened and stiffened shells but forward natural frequencies are first decreased and then increased.
- The number of frequency of fundamental mode is not changed by increasing of angular velocity.
- It is found that the C-section and rectangular section stiffeners are more efficient for stiffening the shells, respectively, for the first mode and the fundamental mode to maximize the defined parameter, $f_{m,n}$.
- It should be noted the modes number is significant for multidisciplinary optimization of

stiffened cylindrical shell under natural frequency and weight constraints.

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