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# A New Backstepping Sliding Mode Guidance Law Considering Control Loop Dynamics

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*In this paper, a new procedure for designing the guidance law considering the control loop dynamics is proposed. The nonlinear guidance loop entailing a first order lag as the control loop dynamics is formulated. A new finite time and smooth backstepping sliding mode control scheme is used to guarantee the finite time convergence of relative lateral velocity. Also in the proposed algorithm the chattering is removed and a smooth control signal is produced. Moreover, the target maneuver is considered as an unmatched uncertainty. Then a robust guidance law is designed without requiring the precise measurement or estimation of target acceleration. Simulation results show that the proposed algorithm has better performance as compared to the proportional navigation, augmented PN and the other sliding mode guidance law.*

**Keyword:** Guidance law, Control loop dynamics, Sliding mode control, Chattering

## Nomenclature

| | |
|---|---|
| $X$ | State Variables Vector |
| $f(x)$ | Nonlinear Function |
| $w(t)$ | Uncertainty |
| $S$ | Sliding Variable |
| $c$ | Weighting Constant in Sliding Variable |
| $u_{eq}$ | Equivalent Control |
| $u_r$ | Reaching Control |
| $t_r$ | Reaching Time |
| $\lambda$ | Line of Sight Angle |
| $\dot{\lambda}$ | Line of Sight Rate |
| $r$ | Relative Range |
| $\dot{r}$ | Closing Velocity |
| $r\dot{\lambda}$ | Relative Lateral Velocity |
| $A_{m,\lambda}$ | Missile Lateral Acceleration |
| $A_{t,\lambda}$ | Target Lateral Acceleration |
| $A_{c,\lambda}$ | Lateral Commanded Acceleration |
| $\tau$ | Time Constant of Control Loop Dynamics |
| $\alpha$ | Bound of Uncertainty |
| $N$ | Navigation Constant |
| $\hat{A}_{t,\lambda}$ | Estimation Of Target lateral Acceleration |
| $\gamma, \beta$ | Controller Gains |

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## Introduction

The basic principle of parallel navigation is to nullify the line of sight rate. In the case of non-maneuvering targets and a zero lag autopilot, proportional navigation guidance law is the optimal solution to the linear homing problem [1], [2].

Using the sliding mode control theory and based on the parallel navigation idea and nonlinear planar engagement kinematics, we can design a guidance law only with information for the maximum target lateral acceleration. Therefore, the precise measurement or estimation of the target acceleration is not required and the proposed guidance law is robust with respect to the target maneuvers [3].

The sliding mode control has been applied to many guidance problems. Zhou et al. proposed an adaptive sliding mode guidance law using linearized equations [4]. Babu et al. [5] studied the guidance law for highly maneuvering targets using the sliding surface of the zero line-of-sight rate based on the Lyapunov method. In [6], for removing chattering of a different sliding surface based on relative range the line of sight rate is defined. In this reference, the commanded acceleration is smoother than the control input in [5], but the calculation is complicated. In [7], based on pure proportional navigation idea, a sliding mode guidance law for delayed LOS rate measurement with sliding variables designed with regard to the relative lateral velocity. In [8], the same sliding variable and true proportional navigation basis is used, but the boundary layer scheme decreases the control precision. A novel sliding mode-based impact time and angle guidance law for engaging a modern warfare ship is presented in [9] and finally in [10], guidance law is designed using the second order sliding mode control.

In all of the above references, the guidance law is designed for a case with ideal dynamics, i.e. no delays exist between the commanded and interceptor applied acceleration. In an actual situation, due to flight conditions and unexpected environmental variables/changes, we cannot expect the ideal performance of the control system. When ideal dynamics are considered in the designing procedure, there is no guaranty for the finite time convergence of the LOS rate and the chance of divergence should be considered. This divergence may severely affect the miss distance and lead to an unsatisfactory performance. To improve the performance, simultaneous design of the guidance and control loop can be used [11, 12].

Sliding mode control theory is known to be robust against parameter uncertainties and external disturbances in nonlinear systems. However, for the sliding surface to be attractive, a switching function must be used in the control law, which causes the chattering of the control signals. The chattering problem in sliding mode control signal is one of the most common handicaps in real applications. Chattering phenomenon is the result of low control accuracy, high heat losses in electrical power circuits and high wear of moving mechanical parts. It may also excite un modeled high frequency dynamics, that degrade the performance of the system and may even lead to instability [13–15].

One approach to eliminate the chattering in control signal is to use a continuous approximation of the discontinuous sliding mode controller called the boundary layer approach. In this method, inside the boundary layer, the discontinuous switching function is interpolated by a continuous function to avoid discontinuity of the control signals.  However, this method brings a finite steady state error and leads to tracking within a guaranteed precision rather than a perfect tracking [16]. The width of the boundary layer is normally constant, and the larger the boundary layer width, the smoother the control signal. Even though the boundary layer design alleviates the chattering phenomenon, it no longer drives the system state to the original state, but to a small residual set around the origin. The size of the residual set is determined by the width of the boundary layer: the larger the width of the boundary layer, the larger the size of the residual set. As a consequence, there exists a design conflict between requirements of the control signals smoothness and the control accuracy. For smoothness of the control signals, a large boundary layer width is preferred, but for a better control accuracy, a small boundary layer width is preferred [16, 17].

The other way is to use higher order sliding mode. HOSM has two important features that make it a better choice in designing the controller. It improves the accuracy of the design, which is a very important issue, and may provide a continuous control [15, 18]. The HOSM generalizes the conventional sliding mode idea, seeking to zero not just the sliding variable, but also some of its time derivatives. In particular, second order sliding modes would provide for the zeroing of the sliding variable and its first time derivative in the finite time, through discontinuous control action acting on its second time derivative, being the sliding variable of relative degree 2 or 1. The problem with this method is that the derivative of a certain state variable is not available for measurement, and therefore methods have to be used to observe that variable and complicated calculations [18, 19].

The back stepping sliding mode control is designed in some references. In [20], an adaptive backstepping sliding mode control is proposed for a class of uncertain nonlinear systems with input saturation. The control law and adaptive updating laws of neural networks are derived in the sense of Lyapunov function, so that the stability can be guaranteed even under the input saturation. This control law is robust against the disturbance, and it can also eliminate the impact of input saturation. [21] presents an integrated missile guidance and control law based on adaptive fuzzy sliding mode control. The adaptive nonlinear control law is designed sing backstepping and sliding mode control techniques. Also in [22] a robust chattering free backstepping sliding mode controller is developed for the attitude stabilization and trajectory tracking control of quad rotor helicopter with external disturbances. The control scheme is developed with the help of backstepping technique and a sliding surface is introduced in the final stage of the algorithm. To attenuate the chattering problem caused by a discontinuous switching function, a simple fuzzy system is used. The asymptotical stability of the system can be guaranteed since the control law is derived based on Lyapunov theorem.

In this paper, a new guidance law is proposed to guarantee the finite time convergence of the relative lateral velocity considering control loop dynamics as the first order lag. The target maneuvers are modeled as bounded uncertainties. For the equation, a new backstepping sliding mode guidance law is designed to guarantee the finite time stability of the system states. Also the finite time convergence is proved using a new sliding condition that leads to decreasing the chattering with high precision in zeroing the sliding variable.

The paper is organized as follows. In section II, the conventional sliding mode control theory is presented and then the equation of motion is formulated. The new back stepping sliding mode control approach is introduced in section III and then the finite time stability of this algorithm is proved. In section IV, the proposed algorithm is used to design the guidance law. Numerical simulation results are shown in section V, and conclusions are reported in section VI.

## SMC Design Algorithm and Plant Modeling

In this section first, the sliding mode control theory is reviewed and then the equation of guidance Loop with approximation of control loop dynamics is formulated.

### Conventional Sliding Mode Control

In this section the conventional sliding mode control is presented [13-15]. Consider a nonlinear system

$$X^{(n)} = f(x) + u + w(t) \qquad , |w(t)| \le \alpha \tag{1}$$

where $f(x)$ is a known nonlinear part, $w(t)$ is a bounded uncertainty, $X = [x \quad \dot{x} \quad \ldots \quad x^{(n-1)}]^T$ is the system state and $u$ is the control input. Then sliding variable is

$$S = \left(\frac{d}{dt} + c\right)^{n-1} \tilde{X} \tag{2}$$

Where $c$ is a strictly positive constant, $\tilde{X} = X - X_d$ and $X_d$ is the desired state. Assume that $u$ in equation (1) must be designed in a way that the following system has the desired properties, and system state reaches the desired amount. The tracking problem for $X = X_d$ is equivalent to making $S = 0$. Conventional first order sliding mode control makes variable $S$ equal to zero in finite time and then maintains the condition $S = 0$ for all future time. Typical sliding mode control consists of a reaching mode, during which the sliding variable $S$ moves to the sliding surface $S = 0$, and a sliding mode, during which the sliding variable is confined to the sliding surface $S = 0$ . Also the sliding variable has no variation from the zero sliding surface in a system without uncertainty. In conventional sliding mode control the control input is designed as follows:

$$u = u_{eq} + u_r , \quad u_r = -kSign(S) \tag{3}$$

Where $Sign(.)$, denoting the Signum function $u_{eq}$ , is the equivalent control determined to cancel the known terms on the first derivation of the sliding variable in system without uncertainty. If there is no uncertainty in the system, $u = u_{eq}$ will maintain the system on the sliding surface. If uncertainties exist, by considering $V = 0.5S^2$ as Lyapunov candidate, a sufficient condition to guarantee the finite time attractiveness of sliding surface $S = 0$ for $S \ne 0$, is to ensure:

$$\dot{V} = S\dot{S} \le -\eta|S| \tag{4}$$

Where $\eta$ is a positive constant, which implies that:

$$t_r \le \frac{|S(0)|}{\eta} \tag{5}$$

The sliding mode controller (3) contains the discontinuous nonlinear function $Sign(.)$. This nonlinearity can cause a chattering problem due to delays or imperfections in the switching devices. Also by selecting the sliding variable as (2) and using the controller (3), the sliding variable reaches the sliding surface in finite time but the state variables are a asymptotically stable.

### Equation of Guidance and Control Loops

In this section, an integrated model for the guidance and control loop is formulated. Guidance and control loops are shown in figure 1, where the outer loop is the guidance loop and the inner loop is the control loop.

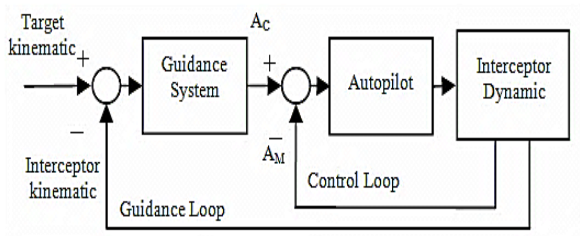


**Fig. 1.** Guidance and Control Loops

Consider a two-dimensional engagement as shown in figure 2, where r is the relative range between target and interceptor, $\lambda$ is the LOS angle with respect to a reference axis, $r\dot{\lambda}$ is the relative lateral velocity between interceptor and target, and $A_{t,\lambda}$ and $A_{m,\lambda}$ represent the target and interceptor acceleration normal to the LOS line, respectively.

The kinematic relation between the target and interceptor is described by a nonlinear equation (6) and the control loop dynamic as a first order lag is described by a linear equation (7) [12, 23].

$$\frac{d}{dt}\left(r\dot{\lambda}\right) = -\dot{r}\dot{\lambda} - A_{m,\lambda} + A_{t,\lambda} \tag{6}$$

$$\frac{d}{dt}(A_{m,\lambda}) = -\frac{1}{\tau}A_{m,\lambda} + -\frac{1}{\tau}A_{c,\lambda} \tag{7}$$

$A_{c,\lambda}$ denotes the acceleration command. The control object is to nullify the relative lateral velocity. Note from equation (6) that $A_{t,\lambda}$ can be treated as the additive unmatched uncertainties of the system.

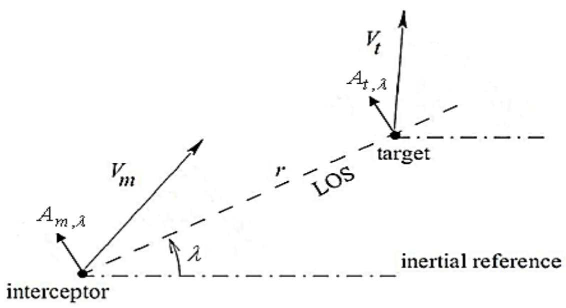


**Fig. 2.** Interceptor - target engagement geometry

## New Sliding Mode Control Algorithm

The sliding mode controller (3) contains the discontinuous nonlinear function $Sign(.)$. This nonlinearity can cause a chattering problem due to delays or imperfections in the switching devices. Therefore, we propose a new sliding mode control by replacing discontinuous Signum function $y = Sign(x)$ by a new continuous function $y = x/|x|^{1-\gamma}$. The responses of the proposed continues function and discontinues signum function are shown in figure 3.

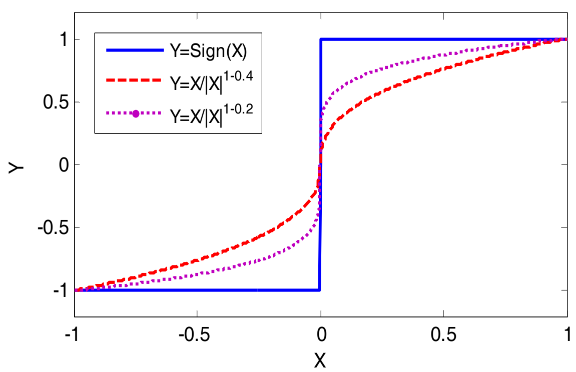


**Fig. 3.** The responses of proposed and signum functions

Since the proposed SMC does not include a signum function, the chattering can be reduced. For this, we use the following theorem.

**Theorem 1**: The reaching term of control input:

$$u_{reching} = -k\frac{S}{|S|^{1-\gamma}}, 0 < \gamma < 1 \quad (8)$$

with (2) as sliding variable and considering $V = 0.5S^2$ as Lyapunov candidate and with satisfying the sliding condition:

$$\dot{V} = S\dot{S} \le -\eta|S|^{1+\gamma} \quad (9)$$

with $k = \alpha|S|^{-\gamma} + \eta$ providing for the convergence of the state $X$ in finite time:

$$t_r \le \frac{|S(0)|^{1-\gamma}}{\eta(1-\gamma)} \quad (10)$$

**Proof**: Substituting equations (8) and (3) in equation (1) and then with substituting the resulted equation in the introduced sliding condition, (9) yields

$$k \ge w\frac{S}{|S|^{1+\gamma}} + \eta \quad (11)$$

By choosing a large enough $k$ in (11), we can now guarantee that (9) is verified. So allowing

$$k = \alpha|S|^{-\gamma} + \eta \quad (12)$$

where $\alpha$ denotes the maximum value of $w$.

For proving (10), let $t_r$ be the time required to hit the sliding surface $S = 0$. Integrating (9) from $t = 0$ to $t = t_r$ leads to

$$\dot{V} = S\dot{S} = S\dot{S}\frac{|S|}{|S|} = |S|\frac{d|S|}{dt} \le -\eta|S|^{1+\gamma} \Rightarrow$$

$$\int_{S(0)}^{0} |S|^{-\gamma} d|S| \le \int_0^{t_r} -\eta dt \Rightarrow -\frac{|S(0)|^{1-\gamma}}{1-\gamma} \le -\eta t_r \Rightarrow \quad (13)$$

$$t_r \le \frac{|S(0)|^{1-\gamma}}{\eta(1-\gamma)}$$

Therefore, (9) is a finite time condition with finite reaching time (10). So, we proved the finite time convergence of $X$. In other words, we have a finite time sliding mode controller.

By substituting equations (12) and (8) into equation (3), control input is given as

$$u = u_{eq} - (\alpha|S|^{-\gamma} + \eta)\frac{S}{|S|^{1-\gamma}} \quad (14)$$

## Guidance Law Design

In this section, the design procedure of the nonlinear guidance law is presented for the integrated guidance and control system given by (6) and (7). We approach the design through a back-stepping sliding mode method. We start with the equation (6). In this equation the control object is zeroing $r\dot{\lambda}$ and $A_{m,\lambda}$ view as the virtual control input. The tools of linearization, exact feedback linearization, Lyapunov redesign, back-stepping, or a combination of them could be used for the design of this virtual control input [13]. To proceed with the design of the new sliding mode control that is proposed in the previous section, set

$$S_1 = r\dot{\lambda} \quad (15)$$

and take the virtual control $A_{m,\lambda_d}$ as

$$A_{m,\lambda_d} = A_{m,\lambda eq} - K_1\frac{S_1}{|S_1|^{1-\gamma}}, 0 < \gamma < 1 \quad (16)$$

For $\gamma = 0$, this acceleration will be discontinuous and oscillation will occur in $S_1 = 0$. Therefore, with a large value for $\gamma$, we have a smooth signal. Equivalent control $A_{m,\lambda_{d\,eq}}$ is chosen to cancel the known terms in the $\dot{S}_1$ equation, that is,

$$A_{m,\lambda_{d_{eq}}} = -\dot{r}\dot{\sigma} \tag{17}$$

A sufficient condition to guarantee the finite time attractiveness of $S_1 = 0$ is to ensure condition of (9)

$$\dot{V}_1 \le -\eta_1 |S_1|^{\gamma+1} \quad ,V_1 = \frac{1}{2} S_1^{\,2} \tag{18}$$

Where $\eta_1$ is a strictly positive constant, which implies:

$$t_{reach} \le \frac{|S_1(0)|^{(1-\gamma)}}{\eta_1 (1-\gamma)} \tag{19}$$

In order to satisfy the sliding condition (18), despite the uncertainty on the dynamics system (6), take

$$\begin{aligned} &\dot{V}_1 = S_1 \dot{S}_1 = \\ &S_1(-\dot{r}\dot{\lambda} - A_{m,\lambda_{d\,eq}} + K_1 \frac{S_1}{|S_1|^{1-\gamma}} + A_{t,\lambda}) = \\ &S_1(K_1 \frac{S_1}{|S_1|^{1-\gamma}} + A_{t,\lambda}) \le -\eta_1 |S_1|^{1+\gamma} \Rightarrow \\ &K_1 \ge \eta_1 + \alpha |S_1|^{-\gamma} \end{aligned} \tag{20}$$

Where $\alpha$ is the maximum value of $A_{t,\lambda}$. Therefore, by replacing equations (20) and (17) in (16), the virtual control or desired interceptor acceleration for nullifying relative lateral velocity yields

$$A_{m,\lambda_d} = -\dot{r}\dot{\lambda} + \left(\eta_1 + \alpha |r\dot{\lambda}|^{-\gamma}\right) \frac{r\dot{\lambda}}{|r\dot{\lambda}|^{1-\gamma}} \tag{21}$$

Now, in the second step we want to design the commanded acceleration $A_{c,\lambda}$ in equation (7) so that testate variable $A_{m,\lambda}$ reaches desired missile acceleration ($A_{m,\lambda_d}$) and track it. Therefore, for this purpose we can introduce the $S_2$ variable as

$$S_2 = A_{m,\lambda} - A_{m,\lambda_d} \tag{22}$$

$A_{m,\lambda_d}$ Is the desired interceptor acceleration for stabilizing $r\dot{\sigma} = 0$ in finite time represented by the right hand of (21). It is clear that if $S_2 = 0$; the variable $S_1 = r\dot{\lambda}$ approaches the origin in finite time that guarantees the intercept. Now we achieve equivalent commanded acceleration $A_{c,\lambda eq}$, while zeroing the derivative $\dot{S}_2$ is given by

$$\begin{aligned} &\dot{S}_2 = \dot{A}_{m,\lambda} - \dot{A}_{m,\lambda_d} = \\ &-\frac{1}{\tau} A_{m,\lambda} + \frac{1}{\tau} A_{c,\lambda} + \ddot{r}\dot{\lambda} + \dot{r}\ddot{\lambda} - \eta_1 \frac{\gamma\left(\dot{r}\dot{\lambda} + r\ddot{\lambda}\right)}{|r\dot{\lambda}|^{1-\gamma}} \end{aligned} \tag{23}$$

To cancel the known term on the right-hand of equation (23), the equivalent control is taken as

$$A_{c,\lambda eq} = A_{m,\lambda} - \tau \left( \ddot{r}\dot{\lambda} + \dot{r}\ddot{\lambda} - \eta_1 \frac{\gamma\left(\dot{r}\dot{\lambda} + r\ddot{\lambda}\right)}{|r\dot{\lambda}|^{1-\gamma}} \right) \tag{24}$$

Let us take the control input as:

$$A_{c,\lambda} = A_{c,\lambda_{eq}} - K_2 \frac{S_2}{|S_2|^{1-\beta}} \tag{25}$$

Note that for $\beta = 0$, this command acceleration will be discontinuous and oscillation will occur at $S_2 = 0$. Therefore, with a large value for $\beta$, we have a smooth commanded acceleration. A sufficient condition to guarantee the finite time attractiveness of $S_2 = 0$ for $S_2 \ne 0$, is to ensure:

$$\dot{V}_2 \le -\eta_2 |S_2|^{\beta+1} \quad ,V_2 = \frac{1}{2} S_2^{\,2}, 0 < \beta < 1 \tag{26}$$

Where $\eta_2$ is a strictly positive constant. In order to satisfy (26), substituting equations (22-24) in (25) yield

$$K_2 \ge \eta_2 \tag{27}$$

By substituting equations (24) and (27) in (25), the commanded acceleration is given as

$$\begin{aligned} &A_{c,\lambda} = A_{m,\lambda} - \tau \left( \ddot{r}\dot{\lambda} + \dot{r}\ddot{\lambda} - \eta_1 \frac{\gamma\left(\dot{r}\dot{\lambda} + r\ddot{\lambda}\right)}{|r\dot{\lambda}|^{1-\gamma}} \right) \\ &-\eta_2 \frac{A_{m,\lambda} - A_{m,\lambda_d}}{|A_{m,\lambda} - A_{m,\lambda_d}|^{1-\beta}} \end{aligned} \tag{28}$$

In this commanded acceleration, measurements or estimates of variables $\dot{\lambda}$, $\ddot{\lambda}$, $r$, $\dot{r}$, $\ddot{r}$ and $A_{m,\lambda}$ are required and the parameters are $\eta_1$, $\eta_2$, $\gamma$, $\beta$ and $\tau$. This command stabilizes the origin $(S_1, S_2) = \left(r\dot{\lambda}, A_{m,\sigma} - A_{m,\sigma_d}\right) = (0,0)$ in finite time. By applying this commanded acceleration, first the interceptor acceleration reaches the desired value (21) in finite time that is adjustable with varying the value of parameter $\eta_2$. Then with varying the value of parameter $\eta_1$, we are able to zero the relative lateral velocity in another desired time.

## Simulation

Numerical simulations are performed to investigate the performance of the proposed algorithm. In this section, we consider situation in which the initial relative distance is 4 km, the closing velocity is 200 m/s, the lateral relative velocity is 230 m/s and the control loop dynamic time constant is equal to 0.5 s.

## Comparison with PN Guidance Family

In this section, the proposed guidance law is compared to the true proportional navigation (TPN) represented by the following guidance command:

$$A_{m,\lambda} = -N\dot{r}\dot{\lambda} \qquad (29)$$

Where the guidance constant is chosen as $N=4$ in this study and augmented proportional navigation (APN) is represented by the following guidance command:

$$A_{m,\lambda} = -N\dot{r}\dot{\lambda} + \hat{A}_{t,\lambda} \qquad (30)$$

Where $\hat{A}_t$ is the estimation of the target acceleration [1, 2]. In this case the target maneuvers with 2 acceleration. We apply the proposed guidance law (28) with $\eta_1 = 2.5$, $\eta_2 = 0.5$, $\alpha = 20$, $\gamma = 0.7$ and $\beta = 0.5$. Note that in comparing the proposed guidance law with PN family, the variables $\ddot{\lambda}$, $\ddot{r}$ and $A_{m,\lambda}$ are required, but it is more robust with respect to the target acceleration.

Fig. 4 illustrates the commanded acceleration and Fig. 5 shows the interceptor acceleration. These figures demonstrate that the maximum magnitudes of the proposed and APN guidance laws are less than that of the PN law and provide good tracking of the target acceleration. Note that in APN, estimation of the target acceleration is used in APN guidance law.

Fig. 6 shows the relative lateral velocity ($S_1$ variable). It is clear from this figure that the relative lateral velocity in the proposed law converges to zero in finite time, but PN is unable to control this variable.

Fig. 7 and Fig. 8 show the relative range, and commanded, missile and desired missile acceleration, respectively. Fig. 9 depicts the interceptor and target trajectory in the proposed guidance law. It is clear from these figures that in the proposed law the missile intercepts with the target in a shorter time in comparison with PN law. Also missile acceleration tracks the desired missile acceleration in the proposed Law.

The integral of the squared acceleration is considered as energy in this paper. Table 1, shows that the interception, the time and the required energy in the proposed law are less than PN. As a result, the proposed guidance law by considering the control loop dynamics in designing procedure has an acceptable performance in comparison with the PN guidance law.

**Table 1.** Intercept Time and Energy

| Guidance Law | Intercept Time (s) | Energy |
|---|---|---|
| Proposed Law | 19.98 | 8570 |
| TPN | 22.6 | 20470 |
| APN | 19.97 | 8770 |

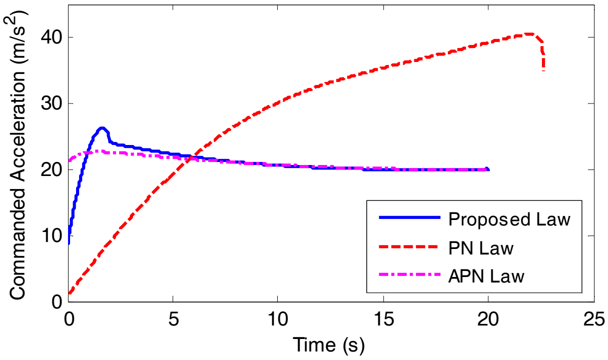


**Fig. 4.** Interceptor Acceleration with different value for $\eta_1$

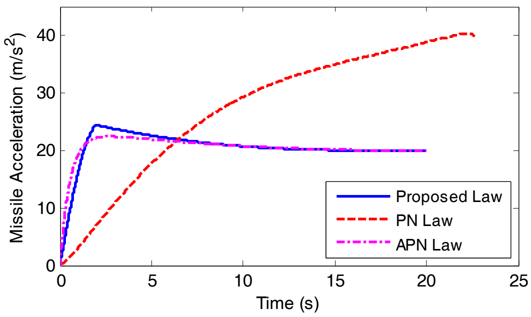


**Fig. 5.** Missile Acceleration

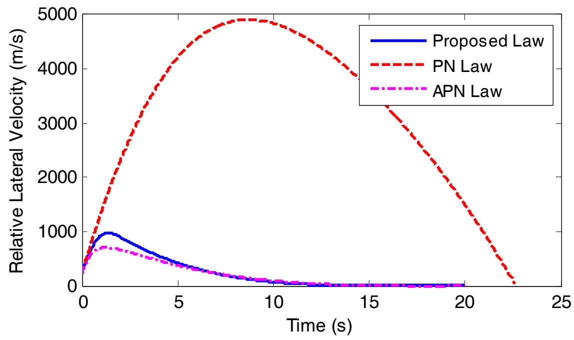


**Fig. 6.** Relative Lateral Velocity ($S_1$ Variable)

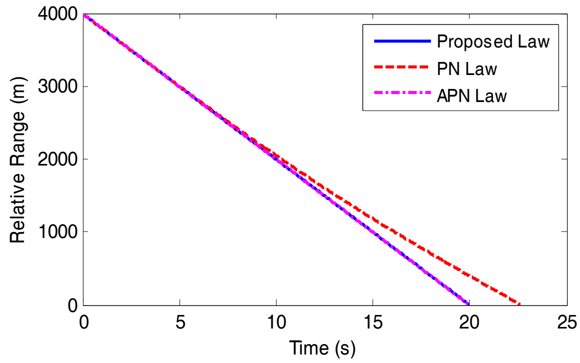


**Fig. 7.** Relative Range

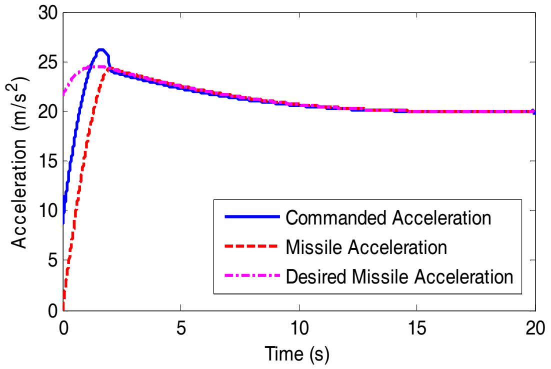


**Fig. 8.** Commanded, Missile and Desired Missile Acceleration in Proposed Guidance Law

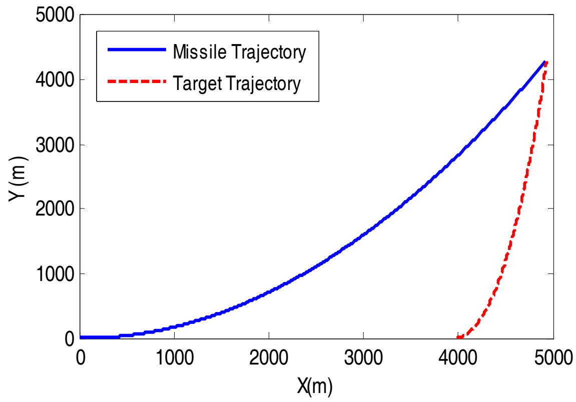


**Fig. 9.** Interceptor and Target Trajectory in Proposed Guidance Law

## Comparison with Conventional SM Guidance Law

In this section, the proposed guidance law is compared to the conventional guidance law designed in [3] and represented by the following guidance command:

$$A_{m,\lambda} = -\dot{r}\dot{\lambda} + \alpha Sign(r\dot{\lambda}) \tag{31}$$

Figures 10 and 11 show the commanded acceleration and the interceptor acceleration. These figures demonstrate that the maximum magnitude of the proposed guidance law is less than that of the conventional sliding mode law and provides a good tracking of the target acceleration.

Fig. 12 shows the relative lateral velocity ($S_1$ variable) and Fig. 13 shows Line of sight rate. It is clear from these figures that the relative lateral velocity and LOS rate in the proposed law converge to zero in finite time, but SM law is unable to control these variables.

Table 2 shows that, the interception time and control effort in the proposed law are less than PN. As a result, the proposed guidance law by considering control loop dynamics in the designing procedure enjoys good performances compared with the PN guidance law.

**Table 2.** Intercept Time and Energy

| Guidance Law | Intercept Time (s) | Energy |
|---|---|---|
| Proposed Law | 19.98 | 8570 |
| Sliding Mode Law | 20.09 | 15700 |

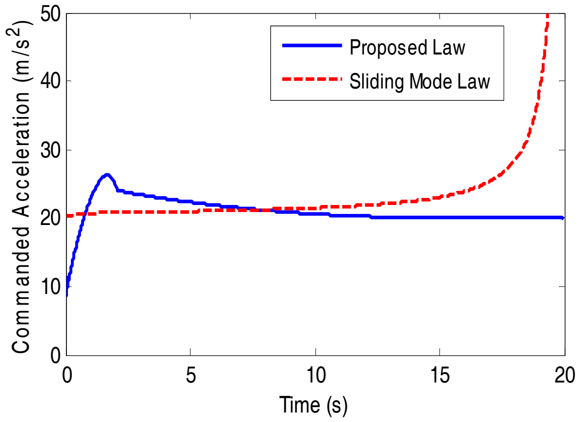


**Fig. 10.** Commanded Acceleration

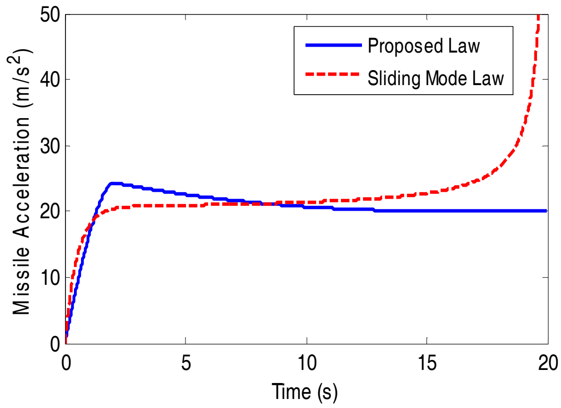


**Fig. 11.** Missile Acceleration

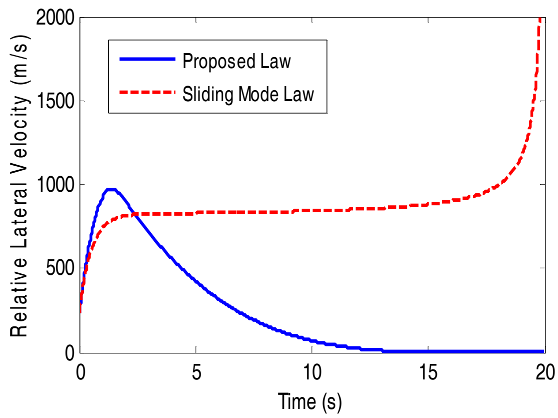


**Fig. 12.** Relative Lateral Velocity ($S_1$ Variable)

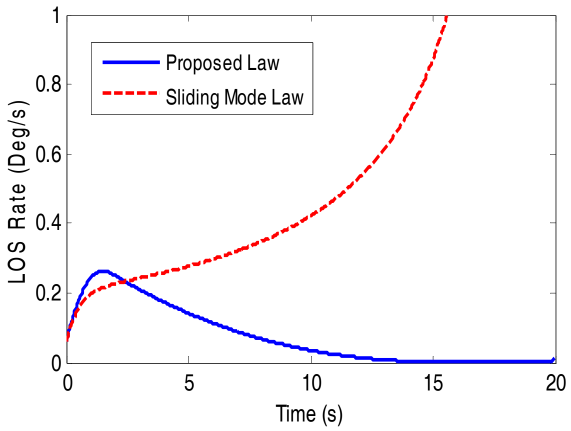


**Fig. 13.** Line of Sight Rate

## The Chattering in Proposed Algorithm

In this section, we apply the guidance law (28) with different values for $\gamma$ and $\beta$ . Note that the $\gamma$ parameter is for adjusting the chattering on the sliding surface 1 (relative lateral velocity). The smoothness in the desired acceleration is generated with relation (21). Also, $\beta$ parameter is for adjusting the chattering on the sliding surface 2 (difference between applied and the desired missile acceleration) and the smoothness of commanded acceleration that is generated using relation (28).

Fig. 14 shows the commanded acceleration and Fig. 15 shows the Sliding variable 2 with $\gamma = 0.7$ and $\beta = 0.2,\ 0.5$. As shown in these figures, as the sliding variable 2 reaches zero with $\beta = 0.2$, chattering occurs, while with $\beta = 0.5$ the commanded acceleration is smooth. Also by increasing the value of this parameter, we are able to adjust the magnitude of the commanded acceleration as well as the time of zeroing the sliding variable 2. Table 3 shows the intercept time and energy with different values for $\beta$ . As seen in this Table, by decreasing the value of $\beta$ , the interception time and the control effort are increased.

Fig. 16 shows the desired missile acceleration and Fig. 17 shows the sliding variable 1, with $\beta = 0.5$ and $\gamma = 0.3,\ 0.7$. As shown in these figures, since the sliding variable 1 reaches zero with $\gamma = 0.3$, chattering occurs, while with $\gamma = 0.7$ the desired missile acceleration is smooth. As seen in table 4, by decreasing the value of $\gamma$ , the interception time and control effort are increased.

**Table 3.** Intercept Time and Energy

| Value of $\beta$ | Intercept Time (s) | Energy |
|---|---|---|
| 0.5 | 19.98 | 8570 |
| 0.2 | 22.25 | 10300 |

**Table 4.** Intercept Time and Energy

| Value of $\gamma$ | Intercept Time (s) | Energy |
|---|---|---|
| 0.7 | 19.98 | 8570 |
| 0.3 | 20 | 10075 |

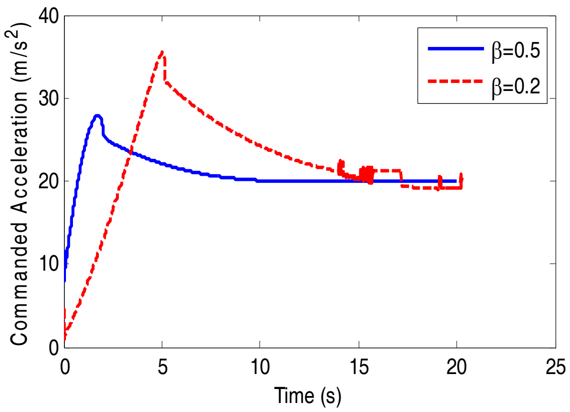


**Fig. 14.** Commanded Acceleration

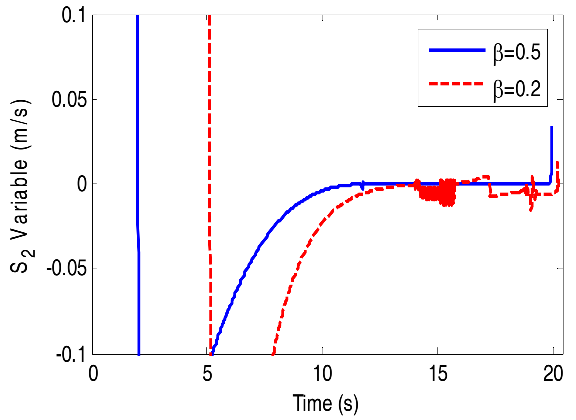


**Fig. 15.** S2 Variable ( $A_{m,\sigma} - A_{m,\sigma_d}$ )

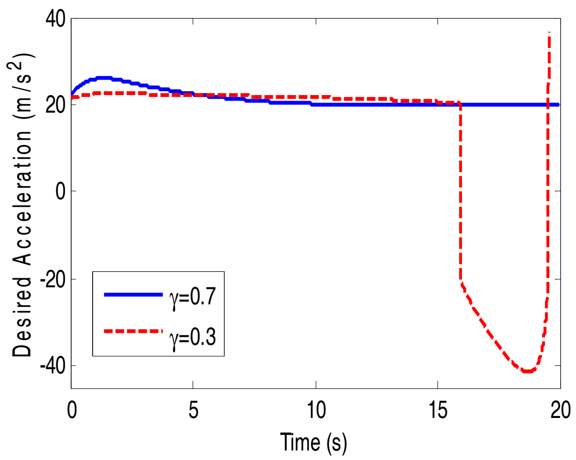


**Fig. 16.** Desired Missile Acceleration

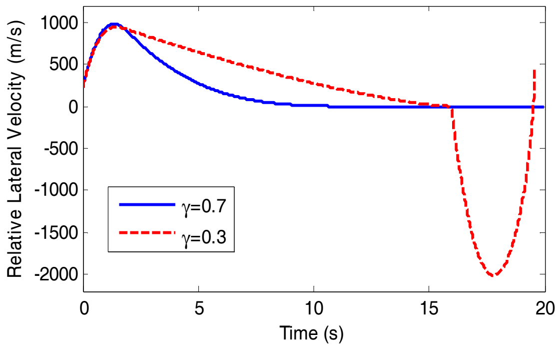


**Fig. 17.** $S_1$ Variable (Relative Lateral Velocity)

## Conclusion

In this paper, the design of a guidance law by considering the first order control loop dynamics is proposed. The proposed algorithm used a new smooth and finite time sliding mode scheme, with a backstepping procedure that guarantees the finite time stability of the integrated guidance and control system. By applying the designed guidance law, first the interceptor acceleration reaches the desired value in finite time that is adjustable. Then with varying the value of a parameter we are able to zero the relative lateral velocity in finite time. It is demonstrated via simulations that a higher performance can be achieved using this guidance law.

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