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Robust Switching Surfaces Sliding Mode Guidance for Terminal Rendezvous in Near Circular Orbit

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The present study aims to present a safe, robust and fast orbital rendezvous guidance. The scheme improves the rate of convergence to equal point until the chaser spacecraft reaches the proximity target. Then, the robust guidance structure is transformed in order to avoid singularity and provide safe rendezvous for reaching the target. Switching is conducted in the guidance scheme by utilizing a self-defined sign function. Moreover, a new modified saturation function is employed instead of the discontinuous part of the sliding mode. The Lyapunov function approach guarantees the asymptotic stability. Numerical simulations are conducted by both linear and nonlinear models of relative dynamics. Mean anomaly, angular velocity, and eccentricity of target orbit are considered as the uncertainties. Finally, the results indicate the performance and robustness of the proposed guidance in the presence of non-linearity, uncertainties, and disturbances, compared to the conventional sliding mode.

Keywords: Rendezvous, Terminal sliding mode, Singularity, Lyapunov function, Rate of convergence

Nomenclature

n	mean motion or constant angular velocity for circular target orbits
a	semi-major axis of orbit
μ	gravitational parameter of the Earth
r_0	radius of nominal circular orbit
$\dot{\sigma}$	Velocityerrors
σ	position errors
ε	boundary layer width

Introduction

Autonomous rendezvous and docking are important parts of some space missions such as assembly of space stations, payload transportation to a space station, and on-orbit servicing of satellites. Generally, rendezvous is one of the key operational technologies for missions that involve more than one spacecraft. Although, the first rendezvous and docking between two spacecraft were done at NASA's Gemini mission, the Soviet space vehicles, Cosmos 186 and 188, did the first automatic rendezvous and docking. Also, Chinese Shenzou-8 and Shenzou-9 spacecraft performed rendezvous and docking with Tiangong-1 space lab successfully [1-3]. One of the first investigations on the rendezvous guidance was accomplished by Aldrin on NASA's Gemini rendezvous mission[4]. Terminal rendezvous is a special part of orbital rendezvous, where relative information is available and the relative distance is small in comparison with the target orbit radius [5,6]. Maneuvers such as Hohmann transfer is not appropriate in about few meters to space station. Due to some reasons like uncertainties and disturbances, even the best executed series of orbital maneuvers might not exactly achieve the appropriate final orbit [7]. Furthermore, the sequence of operations would be relatively rapid when chaser approaches the target. Thus, with regard to availability and accuracy of relative data, the close-loop guidance and control are necessary for the terminal rendezvous [1].

In the recent years, the guidance and control of rendezvous missions has attracted many researchers. state dependent Riccati equation (SDRE) was utilized to control the chaser translational relative motion. Attitude control of the chaser vehicle was achieved by a linear quadratic regulator (LQR) controller. Moreover, a

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linear quadratic Gaussian (LQG)-type control system was proposed by combining the controller with extended Kalman filter [2]. An optimal guidance was introduced for far range rendezvous. Two maneuvers are needed to perform the rendezvous; but due to the open-loop control in the second maneuver, terminal errors can become large. This method is usable for any elliptical orbital rendezvous under constant thrust [5]. Moreover, a GNC system design of a ground test-bed for spacecraft rendezvous and docking experiments was introduced. In order to command the cold gas thrusters and track a mission trajectory profile, the PID control was utilized with Pulse Width Modulators for the chaser [8].

The Lyapunov-based methods are noteworthy for terminal rendezvous because of their robustness and existence of stability proof. A Lyapunov-based robust H∞ controller was designed for the non-cooperative target in circular orbit by assuming the control saturation and uncertainties [9]. A Lyapunov function subject to convex optimization to design a robust control was introduced for spacecraft rendezvous [10].

The sliding mode algorithm as a Lyapunov-based method can guarantee the global asymptotic stability. The sliding mode theory has been studied for many decades as one of the most active areas of research on nonlinear systems theory. The sliding mode is insensitive to and robust against uncertainties and external disturbances. This robust method is characterized by the choice of a sliding manifold in a way that the desired treatment can be achieved by defining the control law. Hence, system states are forced to reach the manifold and remain on it [11-13].

A super-twisting algorithm as a second order sliding mode scheme was designed by contributing nonlinear relative equations and attitude dynamics for spacecraft rendezvous [14]. Also, a terminal sliding mode was designed for rendezvous mission. The terminal rendezvous was assumed to be in libration point between the Moon and the Earth [15]. Binglong and Yunhai proposed a super-twisting controller to rendezvous and docking between two spacecraft [16]. A sliding mode guidance scheme was proposed for spacecraft rendezvous in eccentric orbits with predicting desired states in each time step [6].

The main contribution of this paper is the design of a new switching surface sliding mode guidance for terminal rendezvous of two spacecraft. The target is taken to be in a circular or near circular orbit. The guidance algorithm is designed in a way that chattering can be attenuated more with a higher convergence rate that satisfies the terminal rendezvous constraints. The proposed guidance is based on Lyapunov stability concepts. Though the commands are computed by linearized Hill's equations, the relative motion simulation is based on both linearized and nonlinear original equations.

This paper is presented as follows: Section 2 is dedicated to present dynamics equations of relative motion. The proposed switching surface sliding mode is introduced in section 3. Section 4 establishes the guidance scheme based on the proposed sliding mode scheme for the terminal rendezvous phase. The simulations results are presented in section 5. Finally, concluding remarks are given in section 6.

Model Formulation

The relative motion equations between two spacecraft are important for the closed-loop guidance of terminal rendezvous. These equations are expressed in many astrodynamics texts by approximated linearized equations called Clohessy-Wiltshire equations. These are given with near circular orbit assumption for target spacecraft. The equations are also called Hill's equations. Although these equations are invalid when the distance betweentwo spacecraft is moderately large [3,17,18], the guidance scheme would be designed for the terminal phase of rendezvous; hence, the equations are applicable and useful. The nonlinear relative equations could be derived by substituting the relative kinematics into the Newton's first law and using two bodies gravitational force equation. The relative equations are based on the Cartesian coordinate shown in Fig. 1. The center of coordinate is located at the center of the target spacecraft and z-axis is aligned with the angular momentums direction. The x-axis is in the direction of the position vector from the Earth center to the target spacecraft and y-axis is in a way that the right-handed system of coordinate is completed.

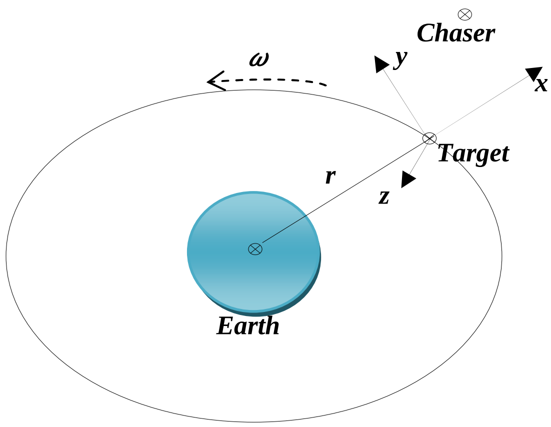


Fig. 1. Coordinate system considered for relative dynamics

The nonlinear equations of motion in absence of disturbances and uncertainties could be presented as follows [19]:

$$\ddot{x} - 2\mathrm{n}\dot{\mathrm{y}} - \dot{\mathrm{n}}\mathrm{y} - \mathrm{n}^2 x = \mu\left(\frac{1}{r_t^2} - \frac{(r_t+x)}{[(r_t+x)^2+y^2+z^2]^{\frac{3}{2}}}\right) + f_x$$
$$\ddot{y} + 2\mathrm{n}\dot{\mathrm{x}} + \dot{\mathrm{n}}\mathrm{x} - \mathrm{n}^2 y = -\frac{\mu y}{[(r_t+x)^2+y^2+z^2]^{\frac{3}{2}}} + f_y \qquad (1)$$
$$\ddot{z} = -\frac{\mu z}{[(r_t+x)^2+y^2+z^2]^{\frac{3}{2}}} + f_z$$

where$n = \sqrt{\mu/a^3}$ denotes mean motion or constant angular velocity for circular target orbits, a shows the semi-major axis of orbit, and μ=GM is the gravitational parameter of the Earth. The x, y, z represent the relative position in the Cartesian coordinate. The term $f_{i=x,y,z}$ refers to the control and perturbing forces per unit of mass in each direction. The linearized equations, so-called the Hill's equations, are [9,17]:

$$\ddot{x} - 2\mathrm{n}\dot{\mathrm{y}} - 3\mathrm{n}^2 x = f_x$$
$$\ddot{y} + 2\mathrm{n}\dot{\mathrm{x}} = f_y \qquad (2)$$
$$\ddot{z} + \mathrm{n}^2 z = f_z$$

There are some errors in determining the orbit due to measurement errors and perturbations. So, the magnitude of semi-major axis of near circular orbit and even nominal circular orbit is not exactly known. So, the uncertainty can be considered in both mean motion and mean anomaly. The uncertainty in the mean motion of the near circular orbit is considered as [9]:

$$n = n_0\big(1 + \varphi(t)\big) \qquad (3)$$

where $n_0 = \sqrt{\mu/r_0^3}$, and r_0 denotes the radius of nominal circular orbit and $\varphi(t)$ expresses the uncertainty as a coefficient of nominal value. Obviously, the uncertainties are bounded and relatively small. In this paper, numerical simulations are performed with the uncertainties due to near-circularity of the target orbit by accounting the uncertainty in eccentricity, mean motion, and mean anomaly.

Nonsingular Sliding Mode with Switching Surfaces

In the rendezvous mission, the relative distances should have no overshoot, and the relative velocity should be sufficiently low or zero. The conventional first-order sliding mode might ensure the relative distance convergence to zero, but not the convergence of its derivatives. In addition, the overshoot problem can occur in this control method. Appropriate changes to the rate of convergence near the target solve this problem. In the sufficient vicinity of the target, the sliding surface switches to obtain the infinite time convergence. This type of convergence could prepare a safe rendezvous condition and get close enough to target in an acceptable time.

In the conventional terminal sliding mode (TSM), the nonlinear sliding surface could fasten the convergence rate. In the first part of this section, TSM is explained. Then, the proposed scheme is introduced to be used for orbital rendezvous guidance.

a) Terminal sliding mode (TSM)

The sliding mode design consists of two major steps. One is the selection of sliding surface; the other is the design of control law which ensures the stability. Because of the special nonlinear surface of TSM, the time of state convergence or rendezvous time could be adjusted [15]. The TSM design is based on a particular choice of the sliding surface and a suitable determination of a control law. The control law forces the states to remain on the surface. When the states slide on the surface, TSM is established and a fast finite convergence is guaranteed. This equation defines a TSM nonlinear surface:

$$S = \dot{\sigma} + \beta\sigma^{\frac{q}{p}} \qquad (4)$$

where β>0, p and q are positive odd integers verifying p>q, and 0.5<q/p<1 [20, 21]. The velocity and position errors are denoted by $\dot{\sigma}$and σ, respectively. To prove the stability with Lyapunov direct method, Lyapunov stability conditions ought to be met. Lyapunov function candidate is chosen as $\mathrm{V} = \frac{1}{2}S^2$. For asymptotical stability, Lyapunov function derivative must be a negative definite (i.e., just when S is zero, this derivative could be zero). This condition is satisfied when:

$$\dot{\mathrm{V}} = \mathrm{S}\dot{\mathrm{S}} = \mathrm{S}\left(\ddot{\sigma} + \beta\frac{\mathrm{q}}{\mathrm{p}}\sigma^{\frac{q}{p}-1}\dot{\sigma}\right) \leq 0 \qquad (5)$$

The difference in TSM method with the conventional sliding mode method would lead to afast response. The sliding manifold is made nonlinear by using states in power. Because of the existence of $\frac{q}{p}\sigma^{\frac{q}{p}-1}$ in sliding manifold derivative (as seen in Eq. (5)), the rate of error elimination is higher than the conventional sliding mode in the vicinity of zero point (equal point) [13]. Also, the finite time convergence is accessable by TSM, while the convergence to the equal point is reached in infinite time by the conventional sliding mode.The negative power appears at derivative of sliding surface. When σ converges to zero, by turning it up in denominator, singularity at the equal point is evident. In addition, in a close vicinity of equal point in the phase plane, the real actuator is saturated and could not satisfy the expected value.

b) Non-singular switching surface sliding mode control

Although terminal sliding mode control in classical form directs towards singularity, the problem will be solved by defining a sliding mode surface with altering the rate of convergence when actuators are saturated. The command threshold condition has the ability to define the time of rate changing or switching the surface. This parameter can be selected by the designer due to mission and safety conditions. Some changes in the nonlinear sliding manifold or the use of input modulators can prevent singularity in TSM. For example, PWPF can be used to realize TSM, and the chaser is finally close to the target less than several meters while the final approach is not achieved entirely. Therefore, the states do not approach zero [15]. In another TSM design, the power is chosen as $q/p \in (1,2)$ for $\dot{\sigma}$; therefore, no negative power appears in the structure and singularity is prevented [22]. It causes loss of the benefits of selecting the power in range of (.5, 1) in the TSM. Suitable switching between sliding surfaces prevents singularity and imparts the advantage of fast convergence at the same time. To hasten the convergence rate in a layer around the equal point, TSM surface is modified. The new surface is defined as:

$$S = \dot{\sigma} + K\sigma^{\frac{q}{p}sign(|\sigma|-1)} \tag{6}$$

where $K > 0$, and 0.5<q/p<1, and the sign function is defined as:

$$sign(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases} \tag{7}$$

Therefore, according to Eq. (6), when the states are on the sliding surface (*S=0*), the system dynamics is:

$$\dot{\sigma} = -K\sigma^{\frac{q}{p}sign(|\sigma|-1)} \tag{8}$$

There are two possible states regarding the dynamics:

$$\begin{aligned} |\sigma| \geq 1 &\Rightarrow sign(|\sigma|-1) = 1\,; \ |\sigma|^{\frac{q}{p}} \geq 1 \\ |\sigma| < 1 &\Rightarrow sign(|\sigma|-1) = -1\,; \ |\sigma|^{-\frac{q}{p}} > 1 \end{aligned} \tag{9}$$

By crossing the lines $\sigma = \mp 1$ in the phase plane, the sign term in Eq. (6) switches the sliding surface. When σis less than one, the relation$|\sigma|^{-\frac{q}{p}} > 1 > |\sigma|^{+\frac{q}{p}}$ is satisfied. In the vicinity of zero point (equal point), the rate of error elimination in the proposed desired dynamics is $\dot{\sigma} = -K|\sigma|^{-\frac{q}{p}}$ and more than TSM by $\dot{\sigma} = -K|\sigma|^{+\frac{q}{p}}$. Although the rate of convergence accelerates, the singularity problem near zero is intensified by the new sliding surface. Another problem is that overshoot can occur and cause collision. It must be ensured that σ and $\dot{\sigma}$ converge to zero smoothly. To solve the singularity problem and use the benefits of the introduced surface, a new switching rule is used. Instead of the sign function, multiple surfaces are defined by means of a new switching function (SF). Therefore, the new sliding surface is modified form of Eq. (6) as:

$$S = \dot{\sigma} + K\sigma^{\frac{q}{p}SF(\sigma,u,U_{thr})} \tag{10}$$

The SF is defined as follows:

$$SF(\sigma,u,U_{thr}) = \begin{cases} 1 & |\sigma| \geq 1 \\ -1 & |\sigma| < 1\,,\ |u| < |U_{thr}| \\ 3 & |\sigma| < 1\,,\ once|u| \geq |U_{thr}| \end{cases} \tag{11}$$

where u is the guidance command, and U_{thr} is the desired threshold. U_{thr}determines when the surface is switched to another one. This function becomes constant once crossing the determined threshold U_{thr} for the time left. This change causes smooth convergence of σ, $\dot{\sigma}$. As a result, the command will reduce due to the reduction of the convergence rate by the switch between surfaces; thus, the singularity in input due to an appropriate switching in the nonlinear power in the sliding manifold can be prevented. In other words, as σ gets closer to zero, the control effort becomes infinite. The sliding surface is switched to another by the infinite time convergence in order to avoid singularity. The new switched surface after input goes beyond the threshold is as follows:

$$S = \dot{\sigma} + K\sigma^{3\frac{q}{p}} \tag{12}$$

where $0.5 < \frac{q}{p} < 1$, and $0.5 < \frac{3q}{p} - 1 < 2$. The power is positive at Lyapunov derivative and singularity is prevented near zero. To further investigate the introduced desired dynamics, supposing that the sliding motion is established ($S = 0$), one could integrate the above equation and the non-trivial solution is easily obtained as:

$$\sigma = \left(K\left(\frac{3q}{p} - 1\right)t\right)^{\frac{1}{-\frac{3q}{p}+1}} + \sigma_0\sigma, t \neq 0 \tag{13}$$

where the relation $-2 < \frac{1}{-\frac{3q}{p}+1} < -0.5$ is simply established. The above equation shows that $\dot{\sigma}, \sigma \to 0$ when t→∞. Therefore, states are forced to slowly converge to zero and the safe rendezvous approach would be achieved.

The power of surface could adjust the rate of convergence to be sufficiently rapid and switching could adapt the rate adequately. Although, in this paper, unity is considered for switching to higher convergence rates, this value could be changed using variable transformation. The transformation could simultaneously maintain the characteristics of the sliding surface and change the switching value.

Inthe next step, in order to achieve the desired dynamics, the guidance commands must be designed by Lyapunov direct method. A proper Lyapunov function candidate could prove the asymptotical stability of the sliding variables. Lyapunov candidate is chosen as in the previous part: $V = \frac{1}{2}S^2$. To satisfy the Lyapunov conditions, the input could be determined in a way that the derivative of Lyapunov function becomes:

$$\dot{V} = -\rho S sign(S) < 0\ ,\ S \neq 0 \tag{14}$$

whereρ is the positive constant. Since this input would include a sign function, chattering effect appears. The function is replaced by saturation function. In order to satisfy the Lyapunov condition (if saturation function is to be used), the following term ought to be obtained in the proposed sliding scheme:

$$\dot{V} = -\rho S sat\left(\frac{S}{\varepsilon}\right) < 0 \quad (15)$$

whereε represents boundary layer width and saturation function is as:

$$sat(x) = \begin{cases} 1 & |x| \geq 1 \\ x|x| < 1 \end{cases} \quad (16)$$

The saturation function could eliminate the chattering, yet, no stability could be proved inside a boundary layer around the sliding surface. Of course, it could be proved that the states converge to the boundary layer about $S = 0$, and stay around the surface once they reach the boundary. However, it is a compromise between the error in the sliding surface and the chattering elimination [23].

TSM idea led to a new approach in the discontinuous part of the input that appeared in Eq. (14). If, instead of S in saturation function, S in the power of p/q is used, the saturation function becomes greater than before in the boundary layer. Therefore, more negative Lyapunov function derivative is gained [13]. This concept could be seen in Fig. 2 for ($S^{0.6}$) as modified saturation function (M. Sat). It means that the discontinuous part modification is more robust against disturbances and uncertainties; furthermore, the sliding scheme is more sensitive to going states off the surface.

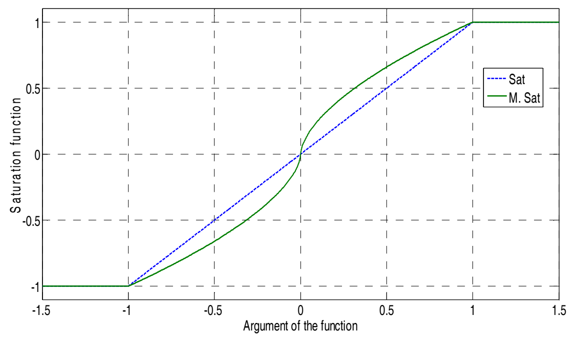


Fig. 2. The common saturation function in comparison with the modified one (sat ($S^{0.6}$))

The commands are derived based on establishing the following equation:

$$\dot{V} = -\rho S sat\left(\frac{S^{\frac{q}{p}}}{\varepsilon}\right) < 0 \quad (17)$$

This new method for chattering elimination has privilege in comparison with the *tanh* (hyperbolic tangent). Because *tanh* could not reach the upper and lower bounds in finite time, while this method is treated as *tanh* and reaches the boundary as a simple saturation function. So, the rate of convergence to the maximum of discontinuous control is increased in the boundary layer of the sliding surface and commands remain continuous. Based on Eq. (16) the input is given in the next section.

Rendezvous Guidance Via Nonsingular Terminal Sliding Mode

A robust sliding mode guidance algorithm based on the proposed sliding surface and the modified saturation function is established in this section. The guidance scheme creates acceleration commands in three dimensions for translational dynamics. The state error is considered to be relative position that must be tending to zero without collision. The sliding surface vector is determined like Eq. (10) for three dimensions:

$$S_i(\sigma_i, u_i) = \dot{\sigma}_i(t) + K_i \sigma_i(t)^{\frac{q}{p} SF(\sigma_i(t), u_i(t), U_{thr_i})} \quad (18)$$

Where $K_{i=1,2,3} = K_x, K_y, K_z$ is considered to be the same for all directions (K), and $\sigma_{i=1,2,3} = x, y, z$. The sliding surface is denoted by $S_{i=1,2,3} = S_x, S_y, S_z$ The Lyapunov candidate $V = \frac{1}{2} S^T S$ should satisfy Eq. (16), where $\boldsymbol{S} = \left[S_x S_y S_z\right]^T$.Eq. (17) is substituted in Eq. (16) as follows:

$$\dot{V} = \boldsymbol{S^T \dot{S}} = \sum_{i=1}^{3} S_i \left(\ddot{\sigma}_i + K\dot{\sigma}_i \left(\frac{q}{p} SF(\sigma_i, u_i, U_{thr_i}) \right) \sigma_i^{\frac{q}{p} SF(\sigma_i, u_i, U_{thr_i}) - 1} \right) \quad (19)$$

Substituting $\ddot{\sigma}_i$ from Eq. (2) for linearized relative motion into Eq. (18) yields the following equations:

$$\begin{aligned} \dot{V} &= S_x \dot{S}_x + S_y \dot{S}_y + S_z \dot{S}_z = S_x \left(f_x + 2n\dot{y} + 3n^2 x + K\dot{x} \left(\frac{q}{p} SF(x, u_x, U_{thr_x}) \right) x^{\frac{q}{p} SF(x, u_x, U_{thr_x}) - 1} \right) \\ &+ S_y \left(f_y - 2n\dot{x} + K\dot{y} \left(\frac{q}{p} SF\left(y, u_y, U_{thr_y}\right) \right) y^{\frac{q}{p} SF\left(y, u_y, U_{thr_y}\right) - 1} \right) \\ &+ S_z \left(f_z - n^2 z + K\dot{z} \left(\frac{q}{p} SF(z, u_z, U_{thr_z}) \right) z^{\frac{q}{p} SF(z, u_z, U_{thr_z}) - 1} \right) \end{aligned} \quad (20)$$

The $f_{i=1,2,3} = f_x, f_y, f_z$ are the guidance acceleration commands. Note that the sliding surface function is piecewise derivable for each amount of SF.To satisfy the Lyapunov condition, an input ought to be given that makes the Lyapunov function derivation negative and satisfy Eq. (16). Hence, the derivation should be as follows:

$$\dot{V} = \sum_{i=1}^{3}[-\rho S_i sat\left(\frac{S_i^{\frac{q}{p}-1}}{\varepsilon_i}\right)] \quad ,i = 1,2,3 \tag{21}$$

The commands of each direction in Eq. (19) are proposed in ways that satisfy Eq. (20). The commands can be given as:

$$\begin{aligned}
u_x &= -2n\dot{y} - 3n^2x - K\dot{x}\left(\frac{q}{p}SF(x,u_x,U_{thr_x})\right) \\
&\quad x^{\frac{q}{p}SF(x,u_x,U_{thr_x})-1} - \rho sat\left(\frac{S_x^{\frac{q}{p}}}{\varepsilon_x}\right) \\
u_y &= 2n\dot{x} - K\dot{y}\left(\frac{q}{p}SF\left(y,u_y,U_{thr_y}\right)\right) \\
&\quad y^{\frac{q}{p}SF\left(y,u_y,U_{thr_y}\right)-1} - \rho sat\left(\frac{S_y^{\frac{q}{p}}}{\varepsilon_y}\right) \\
u_z &= n^2z - K\dot{z}\left(\frac{q}{p}SF(z,u_z,U_{thr_z})\right) \\
&\quad z^{\frac{q}{p}SF(z,u_z,U_{thr_z})-1} - \rho sat\left(\frac{S_z^{\frac{q}{p}}}{\varepsilon_z}\right)
\end{aligned} \tag{22}$$

As for algebraic problems, u_i is replaced with the previous step command $(u_{i_{old}})$ for the switching function. The boundary layers value (ε) depends on the magnitude of the disturbances and uncertainties. The q/p must be determined with regards to the mission and acceptable commands region. Its selection depends on the desired rate of convergence and subsequently the desired time of rendezvous. In spite of the presence of the disturbances and uncertainties, the given commands guide the spacecraft to a safe rendezvous.

Simulation

Numerical simulations are performed for two cases to demonstrate the performance of the proposed guidance using MATLAB software (Mfile& Simulink). The first case deals with linear equations with uncertainties. The second case makes use of nonlinear equations to modeling dynamics. In this case, the non-linearity and uncertainty are exerted to analyze the robustness of the proposed scheme. Also, the target is assumed to be in a near circular orbit at 400 km above the Earth surface.

a) Case1: linear model with uncertainties and disturbance

The rendezvous is performed by three different sliding mode guidance schemes. These schemes are: The proposed sliding mode that is introduced by Eq. (21), the proposed scheme with common saturation function, and the conventional sliding mode guidance with the proposed saturation function. The conventional sliding surface is as:

$$S_i = \dot{\sigma_i} + K\sigma_i \quad , \sigma_{i=1,2,3} = x,y,z, \; S_{i=1,2,3} = S_x, S_y, S_z \tag{23}$$

where the guidance parameters are selected with trial and error as $K = \rho = 1$, $\frac{q}{p} = 0.6$, $U_{thr_{x,y,z}} = 0.5$, $\varepsilon_x = \varepsilon_y = 11.1$, $\varepsilon_z = 0.5$ in all guidance laws. The uncertainties are exerted as: 1% in the mean motion, 2% in the mean anomaly, and eccentricity is perturbed by 0.001. The commands are depicted in Fig. 3, where conventional sliding mode (CSM) and the proposed sliding mode (PSM2) guidance have the new saturation function and PSM1 denotes the proposed control with the common saturation function. Therefore, the effect of the proposed saturation function could be estimated.

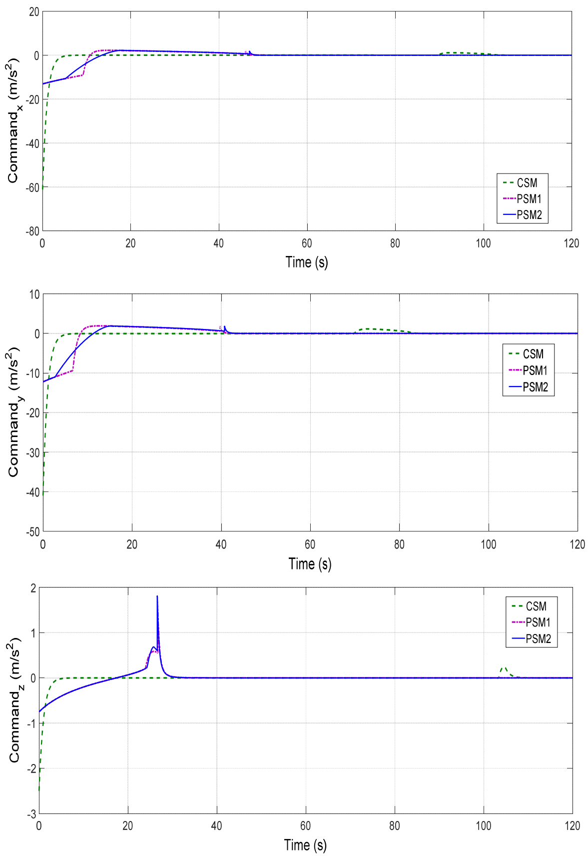


Fig. 3. The guidance commands (case1)

The relative positions and velocities are depicted in Fig. 4 and 5. It could be clearly seen that the states converge to zero in the suitable time. The safe rendezvous is the result of infinite time convergence at the end of the flight. It seems that the entrance to the boundary layer and reduction inthe gain in the saturation function caused a bump after steady states in the guidance commands. Due to a lower rate of convergence of the proposed sliding guidance at the end of the flight, it could be safer than a conventional one; e.g. one could consider the relative velocity in Z direction. The speed is decreased from 0.1 to 0.01 in about 2.3 seconds for PSM2, and in 6.2 seconds for CSM.To show a faster convergence of the proposed sliding mode, the data are given at Table 1 after 70 seconds of simulation. Although the relative position becomes negative, the negligible relative velocity and smooth approach can perform rendezvous sufficiently.

Table 1. Minimum relative distances and velocities at each dimension after 70 seconds

Guidance law	X_{min} (m)	Y_{min} (m)	Z_{min} (m)	$V_{X_{min}}$ $(\frac{m}{s})$	$V_{y_{min}}$ $(\frac{m}{s})$	$V_{z_{min}}$ $(\frac{m}{s})$
PSM2	−0.96	1.40	0.01	-0.018	0.029	-0.0003
PSM1	−0.99	1.42	0.01	-0.018	0.029	-0.0004
CSM	283.6	64.75	17.49	-11.11	-11.05	-0.50

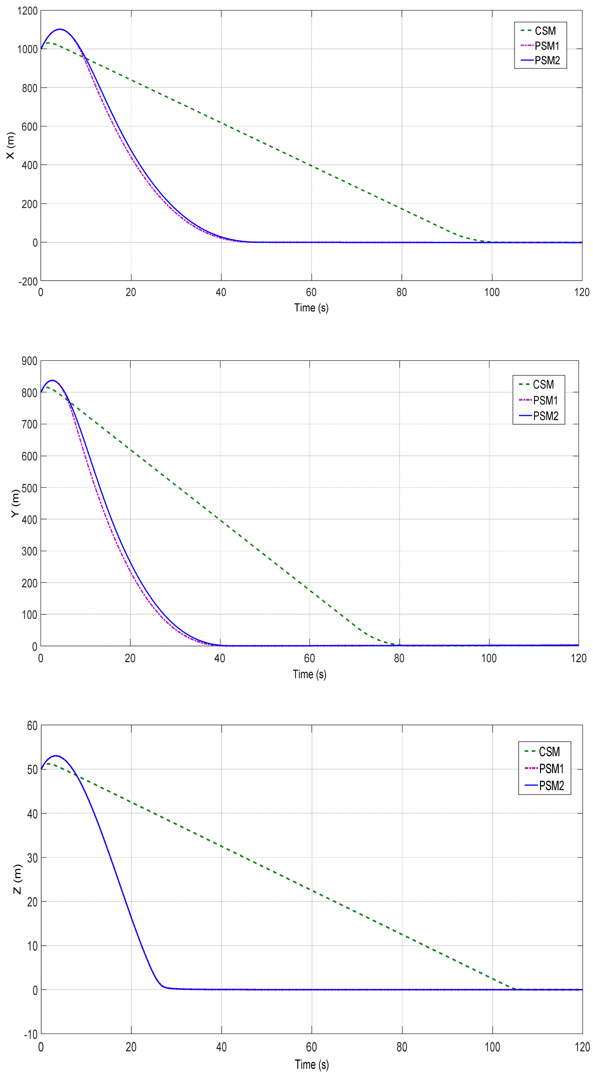


Fig. 4. The relative positions (case1)

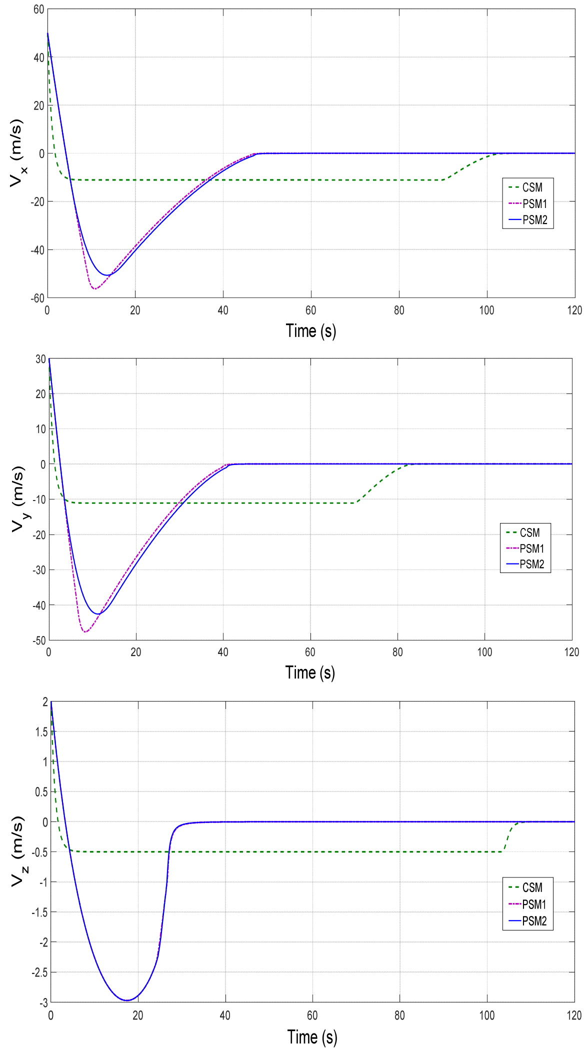


Fig. 5. The relative velocities (case 1)

The singular terminal sliding mode (STSM) that was introduced before as Eq. (6) is performed to show the singularity problem. The command for *x* direction is depicted in Fig. 6. Although the states converge faster than the SF sliding mode, the input becomes singular near zero.

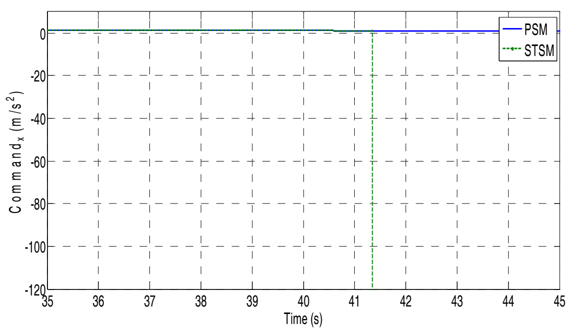


Fig. 6. The singularity in the terminal sliding without switching function

To compare the fuel consumption due to these commands, the ΔVs are represented in Table 2. In this guidance law, a compromise between fuel consumption and the rate of convergence has occurred. Although PSMs have a fast convergence and converge more quickly than CSM, they have greater ΔVs. This rate of convergence during flight can be adjusted by q/p parameter in the switching function. If this parameter is chosen as unity, the guidance becomes CSM. The new discontinuous control (or the new saturation function) helps decreasing ΔV as seen in the given values of ΔVs in Table 2.

Table 2. ΔVsfor terminal rendezvous (case 1)

Guidance law	$\Delta V_x(\frac{m}{s})$	$\Delta V_y(\frac{m}{s})$	$\Delta V_z(\frac{m}{s})$	$\Delta V_T(\frac{m}{s})$
PSM2	151.8	113.4	7.94	272.69
PSM1	163.3	123.0	7.94	294.24
CSM	73.6	53.7	3.00	130.3

In this case, there are several uncertainties; therefore, the use of Monte Carlo simulation might be expected. Monte Carlo simulations are performed for a better demonstration of the performance under perturbation. The eccentricity is supposed to vary between -0.04 and 0.04. Also, Mean anomaly (M) and mean motion (n) change randomly between $\pm 6\%$and $\pm 5\%$, respectively. The mean values of random errors are zero with standard uniform distributions. The magnitudes of minimum distances and minimum velocities at each simulation are represented in Fig. 7. The 200 simulations results show that the proposed methods yield better minimum distances and velocities than the conventional sliding mode. The minimum relative distances magnitude is constricted to lower than 10m for the proposed designs; while it rises to near 20m at the conventional sliding mode. The mean values of relative distances are 7.057m, 3.323m and 3.332m for CSM, PSM1 and PSM2, respectively; also, the standard deviations of these minimums are 4.511, 2.287 and 2.289. The mean values of relative velocities are 0.135m/s, 0.120m/s and 0.121 m/s for CSM, PSM1 and PSM2, respectively. The relative velocities minimum standard deviations are also 0.088, 0.080 and 0.080. These simulations are carried out to show robustness without accounting for the convergence rates. If the simulation time is considered less than 80 seconds, the convergence to zero might not be achieved in the conventional sliding mode. PSM1 and PSM2 guidance are nearly the same because the new saturation function just shows itself at the boundary layer and has not major effects on the minimum relative distances and velocities.

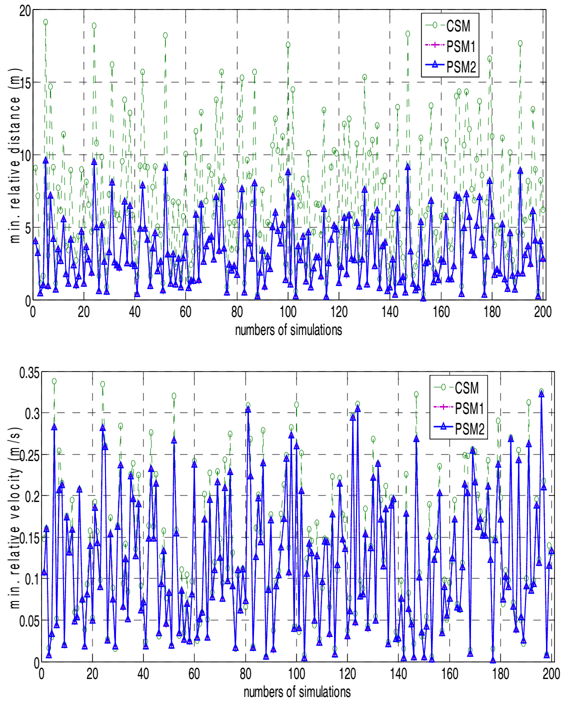


Fig. 7. Monte Carlo simulation results for threen controllers

b) Case 2: Nonlinear model by uncertainty and disturbance

In this part, though Eq. (1) is used as a relative motion equation and the guidance schemes are the same. Now, the dynamics is more accurate and the robustness could be examined due to non-linearity. The designed scheme also deals with the disturbances in three dimensions and uncertainty. Disturbances can be taken as acceleration turned up by gravitational field variation, gravity of third body, accelerometers noises, and the like. These disturbances are considered to be as 1-cosine in x and y directions. To exert disturbance on z direction, a sharp edge disturbance is employed. The disturbances in three directions are shown in Fig. 8. Also, the mean motion (n) is to be with 2% uncertainty. In the previous part, all sliding mode coefficients were the same for all methods. To find out the effect of the proposed nonlinear surface on the fuel consumption with the same convergence time with regard to the common surface, the coefficients of CSM are changed. These changes are as$K = 0.15$and $\rho = 5$. The new scheme can reach faster due to increase in the rate of convergence by utilizing a greater ρ. These coefficents are obtained by trial and error in order to compare the fuel consumptions when both schemes reach the target at approximately same duration of time. The coefficients of proposed schemes are$K = 1$and $\rho = 2.5$. Numerical simulation is executed with the previous case initial position and $\boldsymbol{V_0} = [-5, -3, -2]^T$. The commands are demonstrated in Fig. 9 for three schemes. The relative positions and velocities are shown in Fig. 10, and Fig. 11. The flight trajectory during rendezvous is illustrated for the proposed method in Fig. 12.

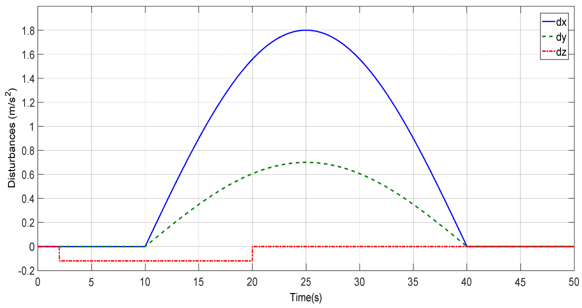


Fig. 8. Disturbances in three dimensions

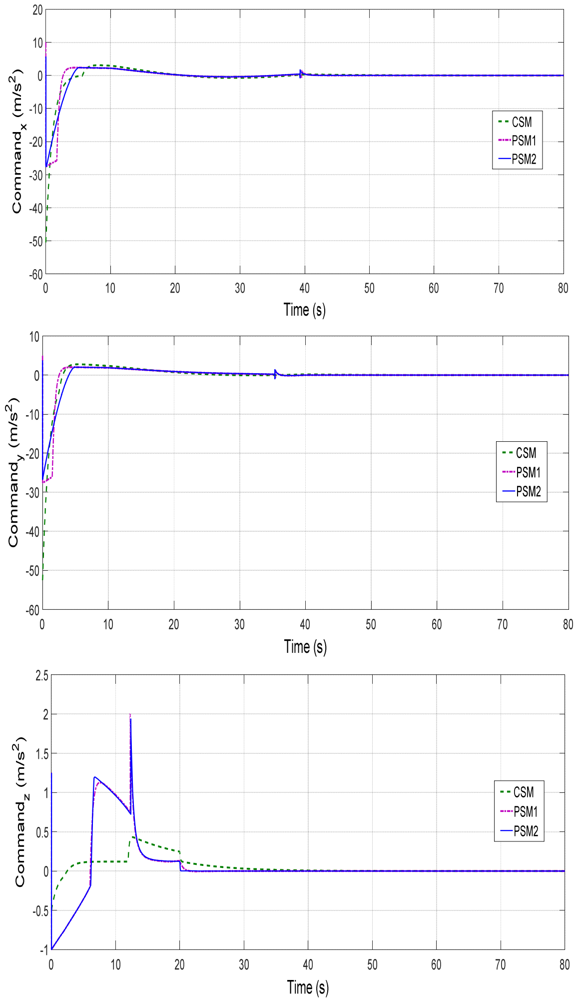


Fig. 9. The guidance commands (case 2)

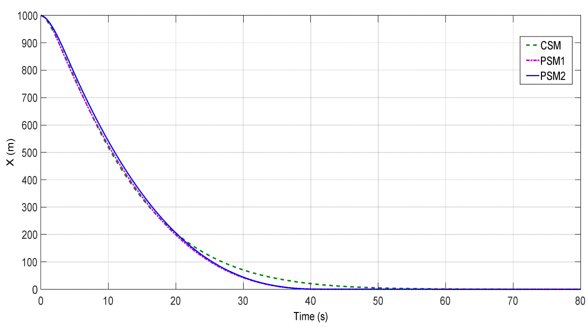
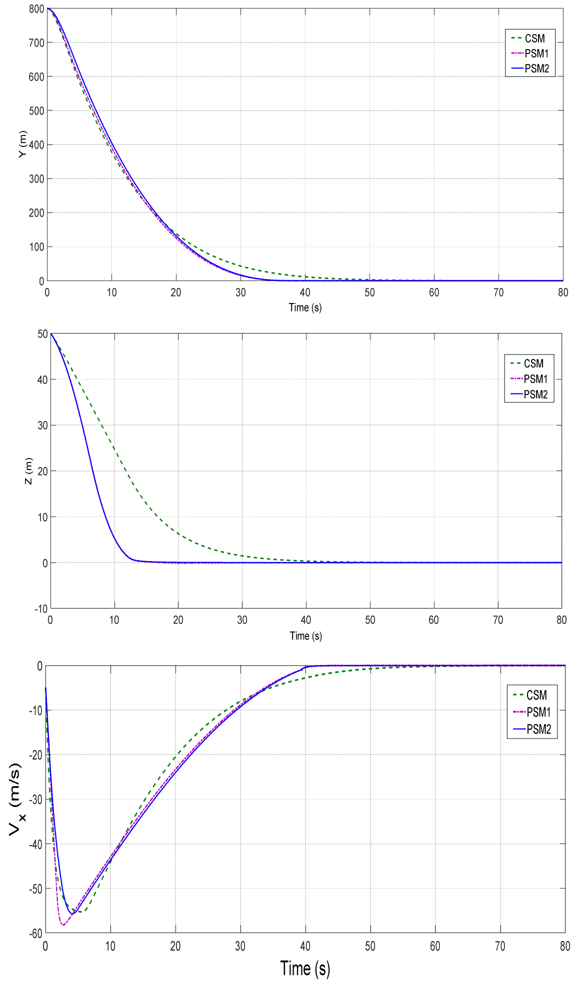


Fig. 10. Relative positions (case 2)



(Continuous) Fig. 10. Relative positions (case 2)

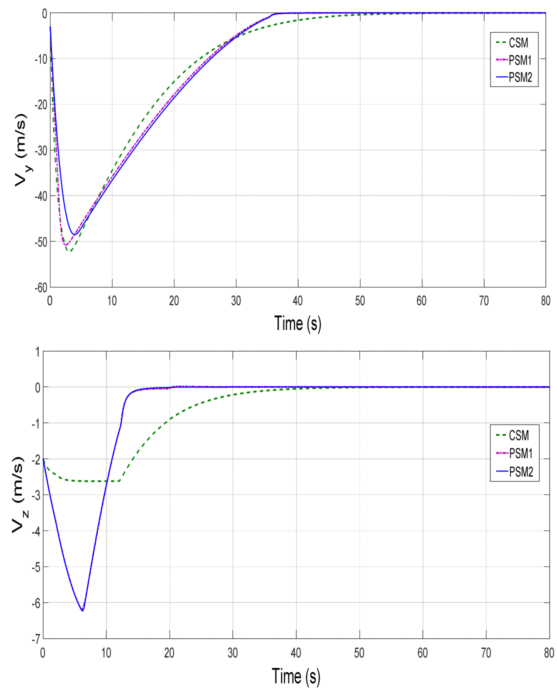


Fig. 11. Relative velocities (case 2)

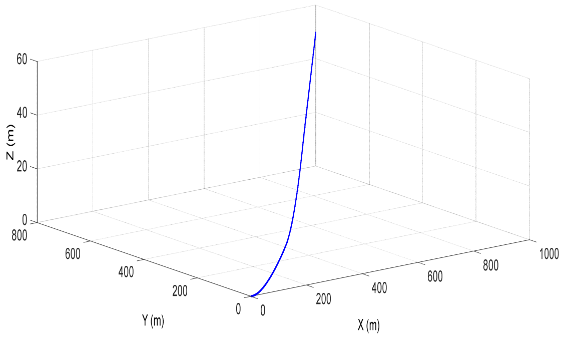


Fig. 12. The trajectory in relative coordinates for the proposed guidance (case 2)

The above figures show that the proposed guidance has a better convergence rate than the conventional sliding surface. Also, the proposed saturation function could improve the rate of convergence near zero and at the boundary layer. Relative distances and velocities after 80 seconds are shown in the Table 3. The PSMs schemes pass zero smoothly due to some disturbances and uncertainty; but the low velocity and rate would not allow violating the safe rendezvous requirements.

Table 3. Relative distances and velocities at each dimension after 80 seconds

Guidance law	X_f(m)	$Y_f(m)$	$Z_f(m)$	$V_{x_f}(\frac{m}{s})$	$V_{y_f}(\frac{m}{s})$	$V_{z_f}(\frac{m}{s})$
PSM2	0.012	0.011	0.006	-3.7e-4	-3.1e-4	-1.1e-4
PSM1	0.012	0.012	-0.006	-3.7e-4	-3.3e-4	1.2e-4
CSM	0.060	0.032	$8.4e-4$	-9.0e-4	-4.8e-4	-1.2e-4

Table 4 shows the fuel consumptions by illustrating ΔV. As discussed for the previous case, the cost for the faster rate of convergence in *z* direction is the higher fuel consumption; this is while the new saturation function reduces the fuel consumption.of course in *x* and *y* directions, the fuel consumption of PSM2 is less than CSM and the rate of convergence is higher.

Table 4. The ΔVs for terminal rendezvous (case 2)

Guidance law	$\Delta V_x(\frac{m}{s})$	$\Delta V_y(\frac{m}{s})$	$\Delta V_z(\frac{m}{s})$	$\Delta V_T(\frac{m}{s})$
PSM2	80.30	79.70	11.64	171.64
PSM1	86.42	83.97	11.61	182.00
CSM	88.49	86.67	5.06	180.22

The minimum relative distance becomes $0.012m$, which is appropriate in this case. Although, the model is linear in designing the proposed guidance; this accuracy might be suitable for berthing and docking considering the low velocities that converge to zero. Therefore, the switching surface is robust and efficient to non-linearity, disturbances and uncertainty. However, without disturbances, uncertainties, and non-linearity, the minimum distance gets closer to 0.03 *m* in 70 seconds that is utterly sufficient for the final rendezvous in docking missions.

Conclusion

The present study develops a new sliding scheme to guide the chaser for terminal rendezvous. The sliding mode guidance scheme is switched by the defined function at the sliding surface. This change in sliding surface allows adjusting the convergence rate and commands. The terminal sliding mode is first utilized. In a layer with unity width of the relative distance, the convergence is accelerated by altering the power. The switching function is defined in order to avoid singularity and provide a safe rendezvous. The function changes the surface to achieve infinite time convergence and approach the chaser smoothly without any overshoots. The prevention of singularity and safe approach could be simultaneously obtained near the target by creating such changes in power. The new saturation function introduced could decrease the fuel consumption as compared with the conventional saturation function at the same boundary layer near the sliding surface. In spite of the proposed plan for a better performance, an increase occurred in the fuel consumption. There is a trade-off between the fuel consumption and a fast response which could be adjusted by changing the power in sliding surfaces.

Monte Carlo simulations were performed to illustrate the robustness and performance of the proposed guidance. In addition, non-linearity effect was investigated based on nonlinear model of motion.Accordingly, different numerical simulations indicated the robustness of the developed guidance in the presence of uncertainties, non-linearity and disturbances. The proposed robust sliding mode could appropriately acquire high accuracy and low relative velocity in the appropriate time.

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