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Flight Path Angle Steering with Lambert Guidance Reference

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In this paper a new guidance technique for ballistic missiles and launch vehicles is proposed. Generally the Lambert guidance is used to generate missile nominal (correlated) parameters through powered flight to put it in a ballistic flight path. Because of uncertainties and undesired factors, the nominal position and velocity obtained by Lambert technique need to be followed in actual flight. In this paper the Flight Path Angle Steering (FPS) procedure is used to accomplish the tracking of nominal parameters. The numerical simulations indicate that the integrated procedure is a cost-effective and suitable scheme for guiding ballistic missiles and launch vehicles especially in design process. In spite of the simplifications made in FPS procedure, numerical simulations show that there is very little difference between the results obtained by FPS and the results obtained by Q-guidance method.

Keywords: Guidance, Flight Path angle Steering, FPS, Lambert problem

Nomenclature

A	Radius of the Earth
a_T	Specific thrust acceleration
D	Drag due to friction
E	Error signal
$\dot{e}$	Error damping
f_a	Specific acceleration
f_g	Specific gravity
f_Z	Specific force acting in the Z direction
G	Gravity
K	Pitch control gain
M	Mass of the missile
r_0	Initial distance from the center of the Earth to the missile
r_F	final distance from the center of the Earth to the missile
T	Thrust value
T_{exp}	Time constant
t_F	Total time of flight
V	Missile current Velocity
V_c	Correlated (Commanded) Velocity
V_g	Velocity to be gained
$\dot{x}$	Velocity component in the earth centered system in the *x* axis direction
$\dot{y}$	Velocity component in the earth centered system in the *y* axis direction
Φ	Central angle
γ	Orientation of the missile velocity with respect to a reference that is tangent to the earth and perpendicular to the vector from the center of the Earth to the initial location of the missile
γ_0	Initial flight path angle
γ_c	Commanded flight path angle or flight path angle of correlated velocity
$\dot{\gamma}$	Actual flight path angle rate
$\dot{\gamma}_c$	Commanded flight path angle rate
μ	Gravitational constant
θ	Pitch angle
θ_0	Initial angular location of the missile with respect to the ***x*** axis of the Earth-centered Cartesian coordinate system
θ_c	Commanded pitch angle
τ	Feedback lead gain
τ_0	Initial feedback lead gain

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Introduction

During the past 50 years, several closed-loop guidance schemes have been developed and successfully employed in a number of launch vehicles and ballistic missiles. Broadly, these schemes can be classified into two categories, namely, path adaptive schemes and perturbation schemes [1-4]. In the case of path adaptive schemes, the steering command is generated from the solution of the simplified equations of motion using the instantaneous state, the desired terminal state, and the system parameters. On the other hand, perturbation guidance schemes assume that the launch trajectory is defined completely before the launch and that the reference nominal trajectory is available. The Q-guidance method is prominent among the perturbation guidance schemes. Battin [5] presents an inspiring historical count of the development of Q-guidance. M. Seetharama [6] has attempted to develop an optimal Q-guidance scheme for a three-dimensional trajectory of a satellite Launch Vehicle. Siouris [7] has described the Q-matrix based on Velocity-to-Be-Gained concept and developed explicit and implicit techniques. Kamal [8] has developed an extended-cross-product steering algorithm based on Q-guidance method.

The elegance of the *Q*-guidance equations lies in the fact that these equations take accelerometer output as a function of time and yield as the output the velocity-to-be-gained V_g which is the difference between the present missile velocity and the velocity required generated by Lambert's routine at that point in space and time for the missile. Thus, if at any point in the powered part of the flight trajectory the velocity-to-be-gained were to vanish, the thrust of the missile could be terminated at that point, and the desired end condition would be realized. Specifically, it is the function of the guidance control system to steer the missile so that the desired cut-off condition in Q-guidance method, $V_g = 0$ will be achieved.

The first drawback of Q-guidance is the Complexity of Q-matrix computation, as the second drawback we can refer to the significant errors most in atmospheric phase of the flight, and the long convergence time of this procedure can be regarded as the third drawback of the Q-guidance method. In this paper we have proposed a new guidance technique called the Flight Path angle Steering (FPS) procedure. The flight path angle steering is a new procedure that lies in the path adaptive schemes. It is assumed that the flight path angle and velocity are obtained by the Lambert procedure which is of interest for many guidance problems [9, 10]. The main problem which is considered in this paper is to propose a guidance scheme based on FPS to follow the values obtained by the Lambert technique. We use a routine based on Zarchan [11] and others [12-16]. These formulae are to be used to compute the velocity and the flight-path angle required at any intermediate time to be compared with the initial velocity and flight-path angle of the missile. As the advantages of the integrated FPS and Lambert Routine, we can refer to simplifying the modeling and computation, Using Inertial Measurement Unit directly and the Short convergence time. In addition, it should be noted that the flight path angle steering procedure can be used to guide missile not only outside the atmosphere, but also at the entire flight. The results of numerical simulations show that the proposed guidance law is simple to implement and it works well for ballistic missiles and launch vehicles; meanwhile the numerical simulations show that in spite of the simplifications made in FPS procedure, the difference between the results of the well-known Q-guidance method and the FPS procedure is negligible. The rest of the paper is organized as follows. In section 2 the essence of the Lambert guidance is introduced. In section 3 the flight path steering control system is proposed. Section 4 incorporates the Lambert guidance technique with the flight path steering procedure. The Flight Mechanics model of the problem is constructed in section 5. Numerical simulation results are presented in section 6 to verify the accuracy and efficiency of the proposed guidance law. Section 7 ends the paper with conclusion.

Lambert Problem

Generally the Lambert guidance is used to control missile through powered flight to put it in a ballistic flight path. Throughout this paper we work under the assumptions of the restricted two-body problem, that is, the interceptor and target are particles of negligible mass above the sensible atmosphere; and it is assumed that the attracting central body has a spherically symmetric gravitational potential. We neglect the interception with other celestial bodies.

With an initial and final radius r_o, r_F, a central angle Φ between them, and a flight path angle γ, sufficient information is available to find the required velocity from the closed-form solution that is fully described by Zarchan [11]. The resultant velocity also can then be used to solve for the flight time from other closed-form solution. Given γ, r_o, and r_F the following relationships, which are based on exact closed-form solutions can be used:

$$\begin{aligned} &\Phi = f(r_0, r_F) \\ &V_c = f(r_0, r_F, \Phi, \gamma_c) \\ &\gamma_c = f(V_c, \Phi, t_F) \end{aligned} \quad (1)$$

r_o, r_F and t_F are given and V_C and γ_C should be found by an implicit manner.

Various methods for solving Lambert's problem have been suggested in e.g. [6-15]. Zarchan [11] has solved the Lambert's problem by fulfilling a speed-up routine which we use it.

In this paper the desired cut-off condition realizes when $V_g=V_c-V<\varepsilon$ will be achieved.

The Flight Path Angle Steering Control System

As we have already mentioned the main problem is to follow the commanded flight path angle offered by Lambert technique. So an error signal proportional to the difference between the commanded and actual flight path angles is produced and enforced to reach zero and a pitch control law is perform using the following control law:

$$\dot{\theta}_c = K(e+\tau\dot{e}) \tag{2}$$

Where, $e=\gamma_c-\gamma$ term introduces error signal, $\dot{e}=\dot{\gamma}_c-\dot{\gamma}$ term introduces error damping, γ_c is the commanded flight path angle of the missile, $\dot{\gamma}=\frac{f_Z}{V}$ is the actual flight path angle rate computed by Navigation Computer, f_Z is the specific force acting in the *Z*-direction. For simulation purposes f_Z is obtained using the equations of motion (plant). K and τ are the pitch control gain and time constant, respectively. Parameter τ is defined as follows:

$$\tau = \tau_0\left(1-e^{-\frac{t-t_0}{T_{exp}}}\right)$$

The initial values of τ_0 and $T_{\exp}$ may be obtained by numerous simulations. Not that τ is an exponential function of time and has been introduced to produce suitably an exponential error signal for stability increasing purpose. This solution is relatively straightforward and simple; the important result is that the accelerometer acts as a low pass filter.

Finally the following control law is obtained.

$$\dot{\theta}_c = K\left[\gamma_c-\left(\gamma+\tau\left(\dot{\gamma}-\dot{\gamma}_c\right)\right)\right] \tag{3}$$

By integrating (3) the commanded pitch angle by which the unit thrust vector of the flight mechanics model is obtained, can be produced. In other words an error signal, that is, a pitch command θ_c that is to be integrated and fed to a missile autopilot that controls missile pitch attitude, can be constructed as (3). Fig. 1 shows the block diagram of the proposed technique.

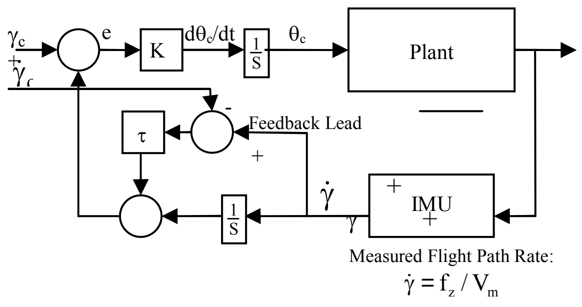

Figure 1. Block diagram of integrating two procedures

Integrating the Two Procedures

In this paper the flight path steering is used to follow the nominal position and velocity values obtained by the Lambert procedure. The nominal values of γ_c obtained by Lambert procedure is based on the initial position, final position and flight time. This parameter and its time rate could be used as input to flight path angle steering procedure as described above. Producing nominal values starts at initial point and continues until the cut-off conditions are satisfied.

Flight Mechanics Modeling

A vector form of the three degree of freedom equations of motion based on the inertial reference frame has been used for simulation purposes. These equations are as follows [7].

$$\dot{\mathbf{r}} = \mathbf{V}$$
$$\dot{\mathbf{V}} = \mathbf{a}_T + \mathbf{f}_g \tag{4}$$

Where $\mathbf{r}=[x,y,z]$ is the position vector of the missile with respect to the earth center, $\mathbf{V}$ is the velocity of the missile with respect to earth, f_g is the specific gravity and a_T is the specific thrust acceleration of the missile. These parameters may be defined as follows:

$$\mathbf{f_g} = -\mu/r^2.\ \text{Unit}\{\mathbf{r}\} \tag{5}$$

$$\mathbf{a_T} = T/m.\ \text{Unit}\{\mathbf{a}_T\} \tag{6}$$

Where $\mu=3.986\times10^{14}\ m^3/s^2$, T is the thrust value and m is the mass of the missile. The direction of the thrust vector is adjusted based upon the pitch status obtained by control law (3).

Numerical simulations

To verify the performance of the integrated procedure, numerical simulations are performed. The algorithmic and graphic engine is SIMULINK MATLAB®. Drag force, the effects of the lateral and forward wind, and misalignment of the launching angle have been studied to investigate the stability. Tables 1 and 2 show the initial and final conditions as well as the missile's characteristics. (Due to security the reference of these data could not be introduced.)

Table 1. Initial and final conditions

R0	6378 km	ϕ_F	5°
V0	0	r_f	6378 m
M0	25000 kg		

Table 2. Weighting, dimensional and propellant values

	Stage 1	Stage 2
Total Weight	25000 kg	11244 kg
Thrust, N	962300	794011
Propellant, kg	11400	6333.3
Burn Time, sec	~ 30	~ 20
Isp, sec	252	250

According to Figure 5, the flight path angle reaches the nominal value obtained by Lambert procedure at cut-off conditions. The commanded pitch coincides with the actual pitch which shows the acceptable operation of the flight path angle steering procedure. The deference between pitch angle and flight path angle is the angle of attack.

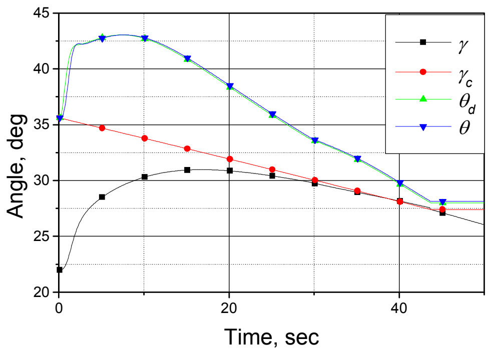

Figure 5. Demanded and actual pitch and flight path angles variations at the first 50 seconds of the flight diagram

As can be seen, the adaptation has been occurred after 48 seconds for Cutoff purposes. If this time defers from Propulsion burnout time, Thrust Termination System (TTS) should be used.

The final gain coefficients for both stages have been obtained as follows.

K=2.5

τ_0 = 3.2 (First & Second Stage)

T_{exp}= 4

These coefficients have been obtained based on good stability and minimizing steady state error by experience and trial and error procedure. Figure 6 shows that the velocity of the missile has reached the demanded (correlated) velocity offered by Lambert procedure.

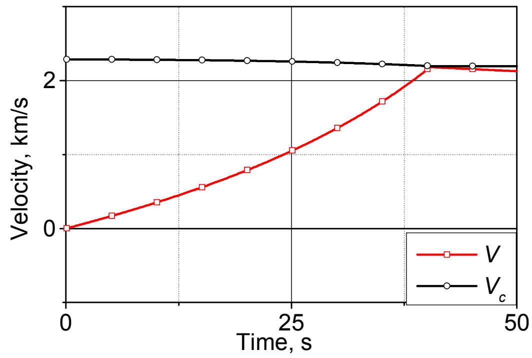

Figure 2. Correlated and actual velocity variations at the first 50 seconds of the flight diagram obtained by FPS procedure

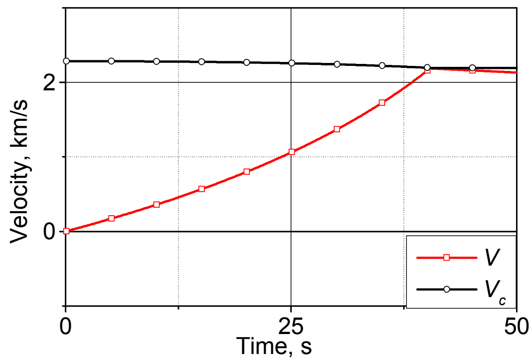

Figure 3. Correlated and actual velocity variations at the first 50 seconds of the flight diagram obtained by Q-guidance method

Figure 3 shows the Correlated and actual velocity variations at the first 50 seconds of the flight diagram obtained by Q-guidance method. As it is seen the difference between FPS and Q-guidance result is negligible, while the FPS's computational complexity is less than Q-guidance method.

Figure 4 shows the flight altitude variations vs. downrange of the missile for the final range angle of 5° (downrange equal to 550 km).

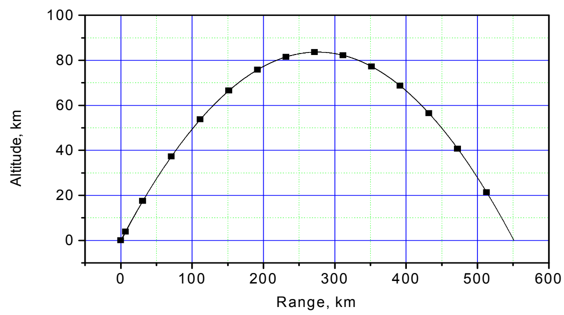

Figure 4. Total altitude variations vs. downrange diagram

Figure 5 shows the flight velocity variations vs. total flight time.

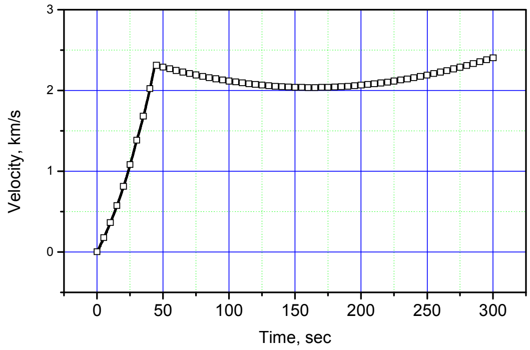

Figure 6. Total velocity variations vs. time of flight diagram

Conclusion

In this paper a new guidance law for ballistic missiles and launch vehicles was proposed. The numerical results of the simulations show that the integrated FPS and Lambert Routine, simplifies modeling and computations required in guidance computer, can use Inertial Measurement Unit directly and has short convergence time, while not only the FPS results are very similar to the Q-guidance results but also its computational complexity is much lower than Q-guidance method. In addition, the flight path angle steering procedure can be used to guide the missile through the entire flight. Moreover, it is suited to be used in design process.

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