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Online Optimal Reentry Guidance via Matched Asymptotic Expansion

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Online optimal reentry guidance of reentry vehicles is the main objective of this paper. The procedure is based on the Matched Asymptotic Expansion (MAE) method, one of the Singular Perturbation Theory (SPT) procedures, and is aided with the Variation of Extremals (VOE) method. The new technique, named MAEOG(Matched Asymptotic Expansion Optimal Guidance) offers a very low solution time and an acceptable accuracy compared with the other numerical methods used until now for reentry optimization. Furthermore, it permits considering both the lift and the aerodynamic roll angle as control variables. The features of the new method appear completely suitable to develop a guidance scheme for atmospheric reentry.

Keywords: optimal control, reentry, MAEOG, solution time

Introduction

The different methods developed inside the Singular Perturbation Theory (SPT) have been extensively used in the last decades in the field of Optimal Control [1],[2] and [3]. Shen [4] demonstrated the usefulness of the SPT procedures in reentry path optimization; Vinh et al. [5] and [6] and Shi [7] and [8] used one of the SPT methods known as matched asymptotic expansion (MAE) to optimize the reentry of a hypersonic vehicle.

This approach presented good and quick solutions and was improved and applied to the reentry problem and other aerospace problems more recently [9], [10], [11] and [12]. In most of these works, the equations were a priori simplified with considering only the lift or the lift to drag ratio as the control. Then, according to MAE method, the general equations were split into outer (Keplerian region) and inner (Aerodynamic-Predominated region) equations, each solved analytically. Finally, the two solutions were matched asymptotically.

This paper presents a method that preserves the advantages of the earlier solutions such as those based only on velocity and in addition permits the optimization considering both the lift and the aerodynamic roll angle as controls. In addition it can be used to develop an online reentry guidance scheme.

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Reentry Model & Problem Statement

The equations of motion of a non-thrusting vehicle entering a planetary atmosphere, assumed to be at rest around a spherical, non-rotating planet are first considered [3](Fig.1A).

$$\begin{cases} \dfrac{dr}{dt} = V\sin\gamma \\ \dfrac{d\theta}{dt} = \dfrac{V\cos\gamma\cos\psi}{r\cos\phi} \\ \dfrac{d\phi}{dt} = \dfrac{V\cos\gamma\sin\psi}{r} \\ \dfrac{dV}{dt} = -\dfrac{\rho S C_D V^2}{2m} - g\sin\gamma \\ \dfrac{d\gamma}{dt} = \dfrac{\rho S C_L V}{2m}\cos\mu - (g - \dfrac{V^2}{r})\dfrac{\cos\gamma}{V} \\ \dfrac{d\psi}{dt} = \dfrac{\rho S {C_L}^2}{2m\cos\gamma}\sin\mu - \dfrac{V^2}{r}\cos\gamma\cos\psi\tan\phi \end{cases} \tag{1}$$

In equations (1), r is the radial distance from the Earth center, θ is the longitude, φ the latitude, V is the absolute velocity, γ is the flight path angle and ψ is the heading. Then the transformed equations are obtained first considering instead of time a new independent variable. It is the elevation above the reference height made dimensionless by it, according to equations (2) and (3):

$$r = r_S + y = r_S(1+h) \tag{2}$$
$$dh/dt = V\sin\gamma / r_S \tag{3}$$

r_S is the reference height and y is the elevation above it. Before introducing h as the new independent variable, one must assume that h is absolutely decreasing during the path. This depends on the initial conditions of the reentry, the initial flight path angle and especially to the lift characteristics of the reentry vehicle.

The Chapman's variables are introduced in the subsequent form:

$$u = V^2\cos^2\gamma / rg \tag{4}$$
$$Z = \frac{\rho S C_L^*}{2m}\sqrt{r/\beta} \tag{5}$$

The subsequent transformations are also considered:

$$B = \frac{\rho_S S C_L^*}{2m\beta} \tag{6}$$
$$E^* = C_L^* / C_D^* \tag{7}$$
$$\varepsilon = 1/\beta r \tag{8}$$
$$C_L = C_L^*\delta \tag{9}$$
$$C_D = C_D^* f(\delta) \tag{10}$$
$$f(\delta) = a\delta^2 + b\delta + c \tag{11}$$

where ρ_S is the atmospheric density at the reference height, S is a reference area from the geometry of the reentry vehicle, β is the reciprocal of the scale height, C_L^* is the lift coefficient corresponding to the maximum lift to drag ratio and C_D^* is the drag coefficient corresponding to the same condition. According to equation (9) , the first control variable, δ can be defined. The other control variable is the aerodynamic roll angle μ ; considering it as a variable, is one of the main differences of the present problem in comparison with other defined previously[8].

For purpose of calculation, $f(\delta)$ is considered as above; empirical data and some references [5] demonstrate that this is an acceptable assumption. The coefficients a, b and c can be allocated considering the special case for which the problem is solved; in other words considering the special aerodynamics of the reentry vehicle treated in each problem.

In addition, instead of $\phi, \psi, and\, \theta$ three new variables are introduced from celestial mechanics. They are $I, \Omega, and\, \alpha$ which according to reference [6] are respectively the inclination of the plane of the osculating orbit, the longitude of the ascending node and the angle between the line of the ascending node and the position vector (Fig.1B). Their relations with the previous set of angles is as follow :

$$\begin{cases} \cos\phi\cos\psi = \cos I \\ \sin(\theta-\Omega) = \dfrac{\tan\phi}{\tan I} \\ \cos\alpha = \cos\phi\cos(\theta-\Omega) \end{cases} \tag{12}$$

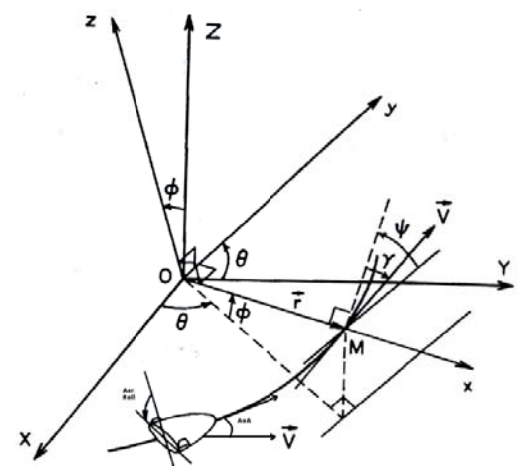

Fig.1 (A) Initial variables and AoA and Aeroynamic Roll shown on the RV

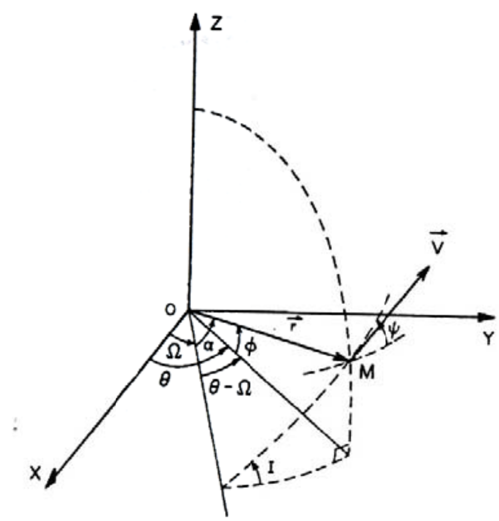

Fig.1(B) Relations between two set of state variables [6]

These leads to the final equations:

$$\frac{du}{dh} = \frac{-u}{1+h} - \frac{2Bu\exp(-h/\varepsilon)\left[\dfrac{f(\delta)}{E^*} + \delta\tan\gamma\cos\mu\right]}{\varepsilon\sin\gamma} \tag{13}$$

$$\frac{d\gamma}{dh} = \frac{\cot\gamma}{1+h}\left(1 - \frac{\cos^2\gamma}{u}\right) + \frac{B\delta\cos\mu\exp(-h/\varepsilon)}{\varepsilon\sin\gamma} \tag{14}$$

$$\frac{d\alpha}{dh}=\frac{1}{(1+h)\tan\gamma}-\frac{B\delta\sin\mu\sin\alpha\exp(-h/\varepsilon)}{\varepsilon\sin\gamma\cos\gamma\tan I} \quad (15)$$

$$\frac{d\Omega}{dh}=\frac{B\delta\sin\mu\sin\alpha\exp(-h/\varepsilon)}{\varepsilon\sin\gamma\cos\gamma\sin I} \quad (16)$$

$$\frac{dI}{dh}=\frac{B\delta\sin\mu\cos\alpha\exp(-h/\varepsilon)}{\varepsilon\sin\gamma\cos\gamma} \quad (17)$$

In these equations δ and μ are the controls and clearly both of them are bounded. The performance measure is defined considering the necessities that usually arise in reentry problems, namely final velocity and coordinates.

$$\begin{aligned} J = & -u(h_f)+ \\ & (1/2)[k_\alpha(\alpha(h_f)-\alpha_d(h_f))^2 \\ & +k_\Omega(\Omega(h_f)-\Omega_d(h_f))^2 \\ & +k_i(i(h_f)-i_d(h_f))^2] \end{aligned} \quad (18)$$

where different k stand for weighting coefficients and must be positive. It must also be considered that h_i *and* h_f are respectively the initial and the final points of the integration.

Boundary Values and Known Coefficients

For the previously stated problem, the boundary known values are considered the initial values of the states in the path. It must be remembered that the initial height h_i is considered almost at the beginning of the dense atmosphere. Finally also h_f is assumed known.

Outline of the Method

At first, using Optimal Control Theory, the optimality conditions are written and the resulting system of differential equations is transformed as mentioned above. The small parameter (which is set zero to singularly perturb the equations) is identified as ε = 1/βr, where the βr, is the reciprocal of the Earth scale height cross its radius. A weighted mean is considered for ε over the Earth atmosphere[5].

At first, with the degeneration (expanding on the basis of ε and considering the zeroth-order terms), the Outer or Keplerian region equations are obtained; they can be integrated analytically.

Then the equations related to the Inner or Aerodynamic-Predominated region (obtained after introducing the new independent variable and expanding for ε and considering the zeroth-order terms) are integrated numerically and the initial conditions for the state equations are obtained with the aid of the matching condition. The matching condition says that the limit of the Inner solution for $h\to\infty$ must equal the value of the limit of the Outer solution for $h\to 0$. The key assumption is that in the initial point, say h_i, the influence of the inner solution on the states is negligible. In other words the assumption is that in the Keplerian region, the influence of the inner or Aerodynamic originated part of the equations is negligible. In this way, equating the initial known conditions with the equations obtained from the solution of the outer equations, the outer solution is completely defined. Then with the aid of the matching condition stated above, the initial value for the integration of the inner equations can be obtained. In other words these initial conditions are almost equal to the value of the inner equations for $h\to\infty$ and so are equal to the limit of the outer (known) equations for $h\to 0$.

The physical insight gained through the state values also permits to make a good guess for the initial values of the co-state equations in the Inner solution. This guess is optimized through the Variation of Extremals iterative procedure [13] and being always near the right solution, few iterations are needed for convergence.

As will be shown, this new technique, which uses the concept of Matched Asymptotic and the capabilities of Variation of Extremals, leads to the solution very fast and has acceptable accuracy.

Outer (Keplerian) Solution

At first, equations (13) to (17) are expanded for ε and the zeroth-order terms are considered. The resulting equations are as follow.

$$\frac{du_o}{dh}=-\frac{u_o}{1+h} \quad (19)$$

$$\frac{dq_o}{dh}=\frac{-q_o}{1+h}\left(1-\frac{q_o^{\,2}}{u_o}\right),\quad q_o=\cos\gamma_o \quad (20)$$

$$\frac{d\alpha_o}{dh}=\frac{1}{(1+h)\tan\gamma_o} \quad (21)$$

$$\frac{d\Omega_o}{dh}=0 \quad (22)$$

$$\frac{dI_o}{dh}=0 \quad (23)$$

The subscripts "o" are showing that these are the outer equations.

Then the optimal procedure is applied. The starting point is the construction of the Hamiltonian matrix. Using it, the co-state equations are derived. In their equations the control-including terms have vanished in the outer equations thus there is no need to solve the optimal control problem. The result is quite logical because in the Keplerian region, (for which the outer equations are written) the aerodynamic controls considered in the problem have negligible effect on the motion.

Then we can find the outer solution analytically.

$$u_o = C_1/(1+h) \quad (24)$$

$$q_o = \frac{u_o}{\sqrt{2u_o + C_2}} \quad (25)$$

$$\alpha_o = \frac{1}{\sqrt{1+C_2}} \tanh^{-1}\left[\frac{u_o - 1}{\sqrt{1+C_2}}\right] + C_3 \quad (26)$$

$$\Omega_o = C_4 \quad (27)$$

$$I_o = C_5 \quad (28)$$

Considering that the assumed h_i is almost on the limit of the dense atmosphere, one can neglect the effect of inner equations on the overall value of the single states. Thus, it is possible to equate the equations (24) to (28) to the initial conditions known for the problem. In this way, the integration coefficients C_1 to C_5 are determined and so the outer part of the answer for the states becomes known.

Inner (Aerodynamic Predominated) Solution

A new independent variable is introduced in equations (13) to (17).

$$\bar{h} = \frac{h}{\varepsilon} \quad (29)$$

After this variation, the resulting equations are expanded for ε and the zeroth-order terms are considered another time.

$$\frac{d\bar{u}}{dh} = -\frac{2B\bar{u}\exp(-\bar{h})}{\sin\bar{\gamma}}\left[\frac{f(\delta)}{E^*} + \delta\tan\bar{\gamma}\cos\mu\right] \quad (30)$$

$$\frac{d\bar{q}}{dh} = -B\delta\cos\mu\exp(-\bar{h}) \quad (31)$$

$$\frac{d\bar{\alpha}}{dh} = -\frac{B\delta\sin\mu\sin\bar{\alpha}\exp(-\bar{h})}{\sin\bar{\gamma}\cos\bar{\gamma}\tan\bar{I}} \quad (32)$$

$$\frac{d\bar{\Omega}}{dh} = \frac{B\delta\sin\mu\sin\bar{\alpha}\exp(-\bar{h})}{\sin\bar{\gamma}\cos\bar{\gamma}\sin\bar{I}} \quad (33)$$

$$\frac{d\bar{I}}{dh} = \frac{B\delta\sin\mu\cos\bar{\alpha}\exp(-\bar{h})}{\sin\bar{\gamma}\cos\bar{\gamma}} \quad (34)$$

The bar shows that these are the inner equations. Then is the turn of the Hamiltonian matrix.

$$\bar{H} = \bar{\lambda}_1\frac{d\bar{u}}{dh} + \bar{\lambda}_2\frac{d\bar{q}}{dh} + \bar{\lambda}_3\frac{d\bar{\alpha}}{dh} + \bar{\lambda}_4\frac{d\bar{\Omega}}{dh} + \bar{\lambda}_5\frac{d\bar{I}}{dh} + \bar{J} \quad (35)$$

Now co-state equations are obtained.

$$\frac{d\bar{\lambda}_1}{dh} = \frac{2B\bar{\lambda}_1\exp(-\bar{h})}{\sin\bar{\gamma}}\left[\frac{f(\delta)}{E^*} + \delta\tan\bar{\gamma}\cos\mu\right] \quad (36)$$

$$\frac{d\bar{\lambda}_2}{dh} = 2B\bar{\lambda}_1\bar{u}\exp(-\bar{h})\left[\frac{f(\delta)}{E^*}\frac{\cos\bar{\gamma}}{\sin^3\bar{\gamma}} + \delta\frac{\cos\mu}{\cos^2\bar{\gamma}}\right] - \frac{(2\bar{q}^2-1)B\delta\exp(-\bar{h})}{\sin^3\bar{\gamma}\cos^2\bar{\gamma}}\left[\bar{\lambda}_3\frac{\sin\mu\sin\bar{\alpha}}{\tan\bar{I}} + \bar{\lambda}_4\frac{\sin\bar{\alpha}\sin\mu}{\sin\bar{I}} + \bar{\lambda}_5\cos\bar{\alpha}\sin\mu\right] \quad (37)$$

$$\frac{d\bar{\lambda}_3}{dh} = \frac{B\delta\sin\mu\exp(-\bar{h})}{\sin\bar{\gamma}\cos\bar{\gamma}}\left[\bar{\lambda}_3\frac{\cos\bar{\alpha}}{\tan\bar{I}} - \bar{\lambda}_4\frac{\cos\bar{\alpha}}{\sin\bar{I}} + \bar{\lambda}_5\sin\bar{\alpha}\right] \quad (38)$$

$$\frac{d\bar{\lambda}_4}{dh} = 0 \quad (39)$$

$$\frac{d\bar{\lambda}_5}{dh} = \frac{B\delta\sin\mu\sin\bar{\alpha}\exp(-\bar{h})}{\sin^2\bar{I}\sin\bar{\gamma}\cos\bar{\gamma}}\left[\bar{\lambda}_4\cos\bar{I} - \bar{\lambda}_3\right] \quad (40)$$

In addition, the optimal controls are obtained according to the Pontryagin Minimum Principle.

$$\bar{H}(\delta^*, \mu^*) \le \bar{H}(\delta, \mu) \quad (41)$$

In this way the optimal controls are equal to their minimum or maximum value or equal to the values indicated by equations (42) and (43).

$$\tan\mu = \frac{1}{\bar{\lambda}_2 + 2\bar{\lambda}_1\bar{u}\cos\bar{\gamma}}\left[\bar{\lambda}_3\frac{\sin\bar{\alpha}}{\sin\bar{\gamma}\cos\bar{\gamma}\tan\bar{I}} - \bar{\lambda}_4\frac{\sin\bar{\alpha}}{\sin\bar{I}\sin\bar{\gamma}\cos\bar{\gamma}} - \bar{\lambda}_5\frac{\cos\bar{\alpha}}{\sin\bar{\gamma}\cos\bar{\gamma}}\right] \quad (42)$$

$$\delta = (1/2a)\Big(-E^*\tan\bar{\gamma}\cos\mu - \frac{\cos\mu\sin\bar{\gamma}E^*\bar{\lambda}_2}{2\bar{\lambda}_1\bar{u}} - \bar{\lambda}_3\frac{E^*\sin\mu\sin\bar{\alpha}}{2\bar{\lambda}_1\bar{u}\cos\bar{\gamma}\tan\bar{I}} + \frac{\bar{\lambda}_4 E^*\sin\mu\sin\bar{\alpha}}{2\bar{\lambda}_1\bar{u}\sin\bar{I}\cos\bar{\gamma}} + \bar{\lambda}_5\frac{E^*\sin\mu\cos\bar{\alpha}}{2\bar{\lambda}_1\bar{u}\cos\bar{\gamma}} - b\Big) \quad (43)$$

On the other hand the maximum and minimum reachable value for δ in each situation depends on the aerodynamic characteristics and performance of the selected reentry vehicle. For the solution of the technical example of this paper we have considered a hypothetical reentry vehicle (shown in Fig.9) and its aerodynamic coefficients for each Mach number and angle of attack are derived by simulation in Fluent 6.1.22 and shown in tables (1) and (2) in look-up table format.

Table1. Variation of E (CL/CD) for different Mach and angle of attacks

Mach / α(deg)	0.5	0.8	1.2	1.5	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.257	0.188	0.103	0.125	0.156	0.189	0.215	0.246	0.247	0.302	0.321	0.342	0.355	0.369	0377	0.382	0.387
4	0.496	0.367	0.210	0.220	0.288	0.373	0.458	0.528	0.582	0.630	0.675	0.721	0.767	0.776	0.781	0.782	0.784
6	0.711	0.533	0.316	0.351	0.467	0.587	0.695	0.820	0.903	0.953	1.003	1.052	1.108	1.116	1.120	1.125	1.128
8	0.897	0.682	0.416	0.457	0.608	0.755	0.919	1.059	1.159	1.202	1.255	1.313	1.378	1.387	1396	1.405	1.411
10	1.050	0.811	0.506	0.577	0.698	0.895	1.114	1.238	1.338	1.387	1.443	1.505	1.577	1.591	1.602	1.610	1.617
12	1.160	0.915	0.583	0.658	0.794	0.997	1.234	1.342	1.453	1.502	1.571	1.637	1.707	1.720	1.730	1.738	1.744
14	1.244	0.996	0.658	0.770	0.890	1.089	1.294	1.387	1.477	1.558	1.647	1.708	1.772	1.783	1.793	1.800	1.806
16	1.295	1.075	0.741	0.808	0.921	1.130	1.317	1.397	1.492	1.599	1.675	1.730	1.786	1.797	1.805	1.811	1.816
18	1.322	1.154	0.832	0.898	0.980	1.150	1.265	1.385	1.471	1.597	1.668	1.716	1.765	1.774	1.781	1.786	1.790
20	1.332	1.218	0.916	0.931	1.000	1.120	1.209	1.357	1.457	1.556	1.635	1.676	1.719	1.726	1.732	1.737	1.741

Table2. Variation of CD for different Mach and angle of attacks

Mach / α(deg)	0.5	0.8	1.2	1.5	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0.124	0.162	0.255	0.208	0.146	0.124	0.113	0.106	0.095	0.090	0.085	0.081	0.076	0.075	0.075	0.074	0.074
2	0.125	0.163	0.256	0.231	0.160	0.130	0.114	0.107	0.096	0.091	0.086	0.082	0.077	0.076	0.076	0.075	0.075
4	0.128	0.166	0.258	0.241	0.180	0.140	0.127	0.111	0.099	0.094	0.090	0.085	0.081	0.080	0.080	0.079	0.079
6	0.134	0.172	0.264	0.246	0.204	0.160	0.135	0.120	0.105	0.101	0.096	0.091	0.087	0.086	0.086	0.084	0.084
8	0.143	0.180	0.272	0.252	0.222	0.177	0.147	0.131	0.114	0.109	0.105	0.100	0.095	0.094	0.094	0.093	0.093
10	0.153	0.190	0.281	0.258	0.233	0.192	0.162	0.146	0.126	0.121	0.116	0.111	0.107	0.106	0.106	0.105	0.105
12	0.166	0.212	0.292	0.265	0.240	0.212	0.190	0.164	0.141	0.136	0.131	0.126	0.122	0.121	0.121	0.120	0.120
14	0.180	0.235	0.305	0.275	0.251	0.235	0.211	0.186	0.158	0.154	0.149	0.145	0.141	0.140	0.140	0.139	0.139
16	0.196	0.251	0.324	0.289	0.266	0.248	0.233	0.211	0.185	0.180	0.171	0.157	0.162	0.161	0.161	0.160	0.160
18	0.214	0.279	0.348	0.318	0.300	0.280	0.264	0.241	0.215	0.200	0.196	0.192	0.188	0.187	0.187	0.186	0.186
20	0.233	0.307	0.378	0.362	0.330	0.310	0.296	0.274	0.243	0.230	0.225	0.220	0.216	0.215	0.215	0.214	0.214

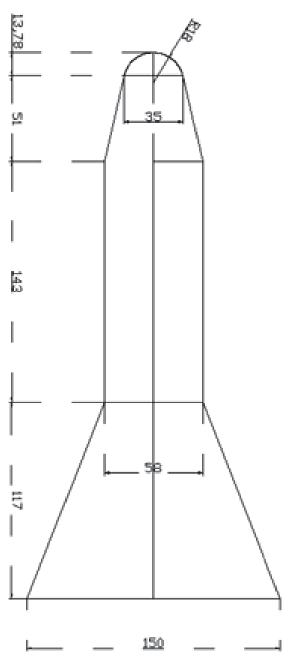

Fig.9 Hypothetical Reentry Vehicle considered for the solution of the technical example

For the aerodynamic roll control the maximun and minimum values are considered as follow.

$$-\pi \leq \mu \leq \pi \tag{44}$$

The analytical solution of equations (36) to (40), applying (41) as a condition, is computationally difficult and so the numerical solution of these equations must be tried. As stated earlier in the outline of the method, the initial value for the integration of the states equation is obtained setting $h \rightarrow 0$ in the outer solutions. The important point to note is that the farther the initial height, the less the error introduced by this assumption. Then the initial value for the co-states must be considered. An arbitrary value is considered for them. Considering that the states are all in the [0 1] neighbourhood an initial value in this space can be a good start. The initial guess is corrected through the known Variation of Extremals procedure. The initial guess can not be completely arbitrary because for initial guess far from the solution, Variation of Extremals can diverge. Now the condition about the final value of co-states must be considered.

$$\lambda^T = -\left.\frac{\partial H}{\partial x}\right|_{h=h_f}$$

$$\begin{cases} \lambda_1(h_f) = & -1 \\ \lambda_2(h_f) = & 0 \\ \lambda_3(h_f) = & k_\alpha(\alpha(h_f) - \alpha_d(h_f)) \\ \lambda_4(h_f) = & k_\Omega(\Omega(h_f) - \Omega_d(h_f)) \\ \lambda_5(h_f) = & k_I(I(h_f) - I_d(h_f)) \end{cases} \tag{45}$$

The weighting coefficients and the desired final values are supposed to be known; for a particular reentry problem the final values are part of the problem; determining the weighting coefficient is not straightforward and right allocation of them needs good insight and experience.

The final value of the states, according to match asymptotic expansion is as follow.

$$X\ (f) = X_o(f) + \overline{X}(f) - X_i(f) \tag{46}$$

Until now, the outer solution is obtained for the states and the intermediate solution can be computed obtaining the limit of the outer solution for $h \to 0$ [6].

$$X_i = \lim_{h \to \infty} X_o \tag{47}$$

With the guess made for the co-states as described, the Variation of Extremals procedure is started. Each time the inner values for the states and the co-states are computed. The outer solution is added to the inner and the intermediate solution according to (46) and the computed co-states are compared with those assumed at the start of each iteration.

Solving a technical example

For a practical example(with (48) as initial points and (49) as desired final point) and a hypothetical reentry vehicle (Fig.9) for which the lift and drag coefficients are computed as functions of Mach number, Reynolds number and angle of attack through Fluent6.1.22 (results in tables 1 and 2), the code written with the mentality of this paper produces interesting results.

$$\begin{cases} h_i = 80km \to h_f = 0km \\ u_i = 0.7399 \\ q_i = \cos\gamma_i = 0.9659 \\ \alpha_i = 0 \quad \deg \\ \Omega_i = 0 \quad \deg \\ I_i = 90 \quad \deg \end{cases} \tag{48}$$

$$\begin{cases} \alpha_d = 2.2918 \quad \deg \\ \Omega_d = 0 \quad \deg \\ I_d = 90 \quad \deg \end{cases} \tag{49}$$

The code converges to the answer rapidly. The entire operation of the code conversion and result addition to the outer and intermediate solutions and forming the global solution lasts 8 ms using a standard 2.4 Ghz speed PC. The accuracy is thought acceptable, because it remains less than 6% compared to a more precise solution method such as Steepest Descent, performed for the global problem. Table 3 resumes the results of this part.

Table 3. Accuracy and solution time comparison between Steepest Descent and MAE

	Final Accuracy	Solution Time
Steepest Descent	Precise	Several Minutes
MAE	6% error	8 ms

As shown by Fig.2, in a typical reentry, the outer solution trend (the starred line) is to slightly increase the velocity but after entering in the dense atmosphere, the power of the inner solution(Aerodynamic-Predominated) overcomes those of the outer solution and obliges the velocity to decrease. Fig.3 shows the result for the Path angle. Fig.4 makes clear that the error of the method compared with Steepest Descent is quite negligible. Fig.5 is an exaggerated view of Fig.4. Fig. 6 shows the optimal controls.

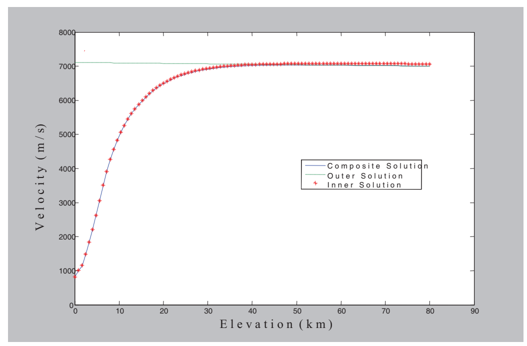

Fig.2 Velocity across Height and the role of outer and inner solution

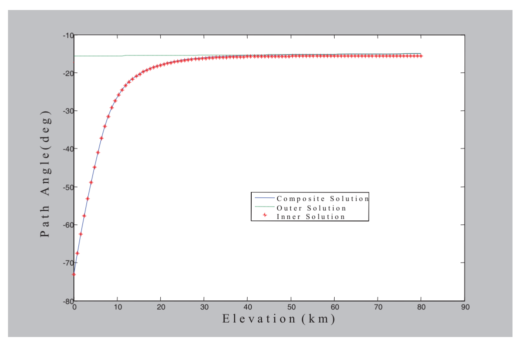

Fig.3 Path Angle across Height and the role of outer and inner solution

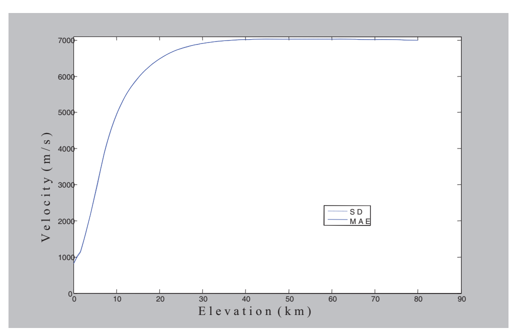

Fig.4 Comparison between Steepest Descent and method presented

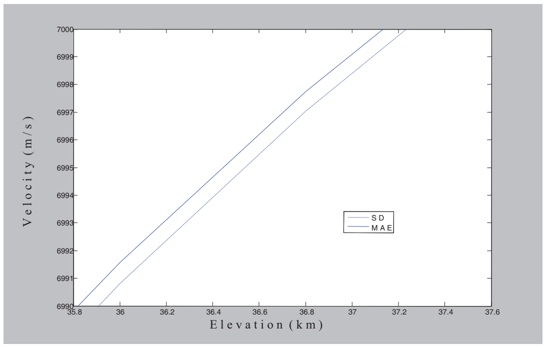

Fig.5 Exaggerated view of Fig.4

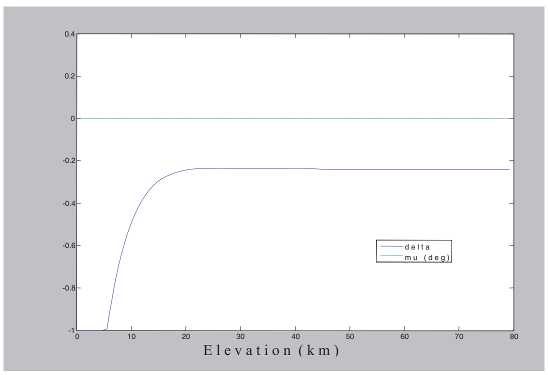

Fig.6 Optimal Controls

The results, particularly those related to velocity across height and path angle are also compared with some previous calculations [15] and they seem quite acceptable.

Implementation as a Guidance Scheme, MAEOG Method and Perturbations Considered

Singular perturbation methods were previously used to generate online the optimal trajectories for aircrafts [16], [17]. This earlier works suggest a similar approach for the reentry case. In fact, instead of considering a guidance scheme that tries to make zero the error of the path respect to a pre-known path, the guidance scheme developed on the basis of the method presented in this paper, can compute online the "new" optimal trajectory at each new point of the trajectory that can be out of the initial optimal path. In other words, the reentry vehicle during its motion, receives with a frequency that depends on the time of the computation of the optimal path (say, around 125 Hz for our problem), new control inputs for flying on the new optimal path, obtained with the data transmitted to the onboard computer the last sampling time (8 ms in the past) Fig.7. The result is the method that is called MAEOG.

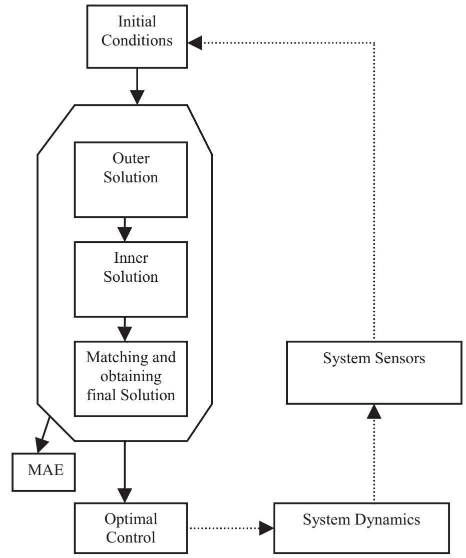

Fig.7 Implementation of the method as an online guidance scheme (MAEOG)

Different phenomena can influence the motion of the vehicle and cause its deviation from the theoretical conditions. The high thermal heating can cause the melting of some parts of the thermal protection, therefore change the lift, and drag coefficients.

In Fig.8, is assumed that something in the flight influences the lift reducing it. This change creates 13% error in the final latitude achieved but by the guidance scheme described, this error is reduced to 7%.

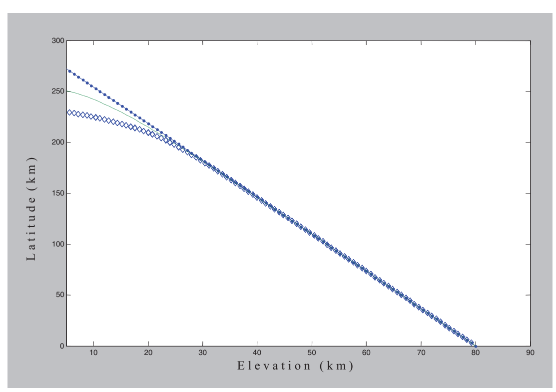

Fig.8 Comparison between the guided path (MAEOG) and the un-guided path

The other great source of disturbance is the atmospheric turbulence and the wind.

Another point to consider is the sensitivity of the method to variations in the initial conditions from which the computation is started. In other words, how can the vehicle correct the initial errors respect to a predefined path if crossing a specific point in the space is the purpose of the flight. Also this analysis can be done by simply generalize the MAEOG method.

It cannot be ignored that in the online guidance sheme, after each time step, the guidance code needs to receive the new coordinates, velocity and path angle to start another computation and optimal path generation.

Providing the reentry vehicle with this data can be done with the aid of sensors, earth bases and satellite systems.

Results and Conclusions

The new technique presented in this paper optimizes the atmospheric reentry trajectory with a considerably good velocity. For typical initial values, the code written on its base leads to the results in less than 10 ms with a normal PC with 2.4 Ghz speed. Moreover, its precision is comparable with the other frequently used methods such as Steepest Descent or Multiple Shooting. The Optimal Controls obtained for the hypothetical reentry vehicle of the paper are shown. Considering the features of the method, it can be considered a valuable candidate for the development of an efficient online reentry guidance scheme, also presented here and named MAEOG considering the computational basis of it. It has almost the precision of the known valid methods with the difference that it is faster and its implementing needs simplest hardware, since using simplest mathematical relations.

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