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Finite Time Guidance Law to Intercept Desired LOS Angle Using NTSM Control

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Nonsingular terminal sliding mode (NTSM) guidance for intercepting the desired line of sight (LOS) angle in terminal phase is proposed in this paper. In order to satisfy the predefined LOS angle and to intercep into target, a nonsingular terminal sliding variable is introduced. In reaching phase, in the presence of uncertainties such as target maneuvers, robust NTSM guidance law is designed in order forzeroing the sliding variable in finite reaching time. Then, in sliding phase, due to introducing nonsingular terminal sliding variable, finite time stability of line of sight angle and line of sight angular rate is granteed without singularity in commanded acceleration as control signal Numerical simulations are presented to illustrate the potential of the proposed guidance law.

Keywords: Guidance law; Impact angle, NTSM control, Parallel navigation

Nomenclature

X	State Variables Vector
$f(X)$	Nonlinear Function
S	Sliding Variable
λ	Weighting Constatnt in Sliding Variable
$u_{reaching}$	Reaching Control
$t_{reaching}$	Reaching Time
q	Line of Sight Angle
$\dot{q}$	Line of Sight Rate
R	Relative Range
$\dot{R}$	Closing Velocity
A_M	Missile Lateral Accelaration
A_T	Target Lateral Accelaration

Introduction

The proportional navigation guidance law is one of the most widely employed strategies in tactical homing interceptors. The basic principle of PN is to nullify the line of sight rate based on parallel navigation idea. In the ideal case of non-maneuvering targets and zero lag autopilot, PN is an optimal guidance law, but it is not capable ofcontrolingthe impact angle [1], [2].

Using sliding mode(SM) control and based on parallel navigation, we can design a nonlinear and robust guidance law for target maneuvers [3]. This theory has been applied to many guidance problems. In [3-7],first order SM guidance laws and in [8] and [9], guidance laws design using second order SM control were studied.

When using a missile to intercept a target, the impact angle is often important. In particular, in order to increase the effectiveness of warheads against some targets such as tanks, a specific impact angle is required. Therefore, a guidance law which can guarantee a specified impact angle is very desirable [10]. Impact angle guidance laws have been presented in the literature. In [11], an impact angle guidance law is achievedby numerically solving the two point boundary value problem. In [12], the authors analytically derived a bias termfromthe PN guidance law to account for impact angle constraints. Adaptive guidance laws for obtaining specified impact angles and applying them to the reentry guidance of a hypersonic gliding vehicle is introduced in [13]. An integrated guidance and control approach to solving the impact angle problem through a backstepping method is presented in [14]. In [15], optimal control theory with a weight on the time-to-go is employed to solve the guidance problem. The combination of Lyapunov stability theory and a parameter optimization approach areutilized in [16]. In [17], the impact angle problem using a SDRE approach, where

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the state weighting matrix is assumed to be a function of the time-to-go, is solved.

Because of nonlinearity and robustness properties of SM control theory, SM guidance law for intercepting thespecified angle is desirable. In [18], the authors presented conventional SM control theory based guidance laws to intercept non-maneuvering targets at a desired impact angle. The desired impact angle, defined in terms of a desired line of sight angle, is achieved by selecting the missile lateral acceleration to enforce sliding variable on a conventional sliding surface based on LOS angle and its rate. A three-dimensional guidance law with angle constraint based on conventional SM control is designed in [19].

In conventional SMC, the most commonly used sliding variable is the linear which is based on linear combination of the system states by using an appropriate time-invariant coefficient. Then,the SM controller is designed which drives the system to reach and remain on the linear sliding surface in finite reaching time. Except for sliding phase in which the transient response can be made faster by utilizing a larger valued coefficient in the linear sliding variable, the system states cannot converge to the equilibrium point in finite time. In other words, an arbitrary linear manifold is considered a sliding variable which can guarantee the asymptotic or exponential stability in sliding phase. Therefore, the main disadvantage of conventional SMC is that the system states cannot reach the equilibrium point in finite time [20, 21]. Recently, finite time stability and finite time control have been constructed for some systems. Finite time stable systems might enjoy not only faster convergence but also better robustness and disturbance rejection properties [22]. Instead of a linear sliding variable, there exists a terminal sliding mode(TSM) control with a nonlinear sliding variablewhich grantees finite time stability in sliding phase. The TSM was developed by adding the nonlinear fractional power item into the SM to offer some superior properties, such as finite time convergence as well as faster and better tracking precision [21, 23]. TSM and finite time stability theories are used for designing impact angle guidance laws in the literature [10, 24-26].

However, there is an intrinsic singular problem in TSMC due to using fractional power functions as the sliding variable. The singularity problem limits the applications of the TSMC and therefore, NTSM method is proposed to avoid that singularity[27, 28]. In [27], a method to overcome the singularity problem of TSM control systems isproposed and in [28], a non-singular fractional TSM control is presented. Also, in [29], Non-singular TSM Guidance and Control with Terminal Angle Constraints is explained. However,this guidance law is designed for intercepting non-maneuvering targets.

In this paper, a newguidance law is developed using NTSM control theory to intercept desired LOS angle. By applying the proposed guidance law, the finite time convergence of the LOS angle to desired value and its rate to zero in the presence of uncertainty such as target maneuvers are guranteed. First, the SM controller can curb the state trajectory to the nonsingular nonlinear terminal sliding surface in finite reaching phase time and then because of introducing the particular nonlinear sliding surface, the state variables converge to equilibrium point in finite sliding phase time without singularity in commanded acceleration as control signal. For removing the chattering in commanded acceleration, continuous approximationmethod is used.

The paper is organized as follows: In Sec. II, the SM control theory is introduced and then modeling of relative kinematic will be elaborated. Sec. III proposes the new guidance law. Numerical simulation results are shown in Sec. IV and conclusions are presentedin Sec. V.

SM Control

Consider a nonlinear system:

$$X^{(n)} = f(X) + u \tag{1}$$

$$f(X) = f_{nom} + f_{un}, \quad |f_{un}| \le \alpha \tag{2}$$

Where f_{nom} is a known nonlinear part, f_{un} is a bounded uncertainty, $X = [x \quad \dot{x} \quad \dots \quad x^{(n-1)}]^T$ is system state and u is the control input. In SM scheme, first a sliding variable is introduced using system errors. Typical SMC consists of a reaching mode, during which the sliding variable moves to the sliding surface, and anSM, during which the sliding variable is confined to the sliding surface and has no variation from sliding surface [30-32]. Therefore, designing SM controller is presented in section (A) in reaching phase and then in sliding phase, the sliding variable is introducedin section (B).

Reaching Phase

In reaching phase, SM controller is designed for zeroing sliding variable or reaching sliding surface (S=0). SM controller makes S equal to zero in finite reaching time and then, maintains the condition $S = 0$ for all future time. In conventional SMC, the control input is designed as follows:

$$u = u_{eq} + u_{reach} \tag{3}$$

u_{eq} is the equivalent control determined to cancel the known terms on the first derivation of S in system without uncertainty and u_{reach} is the reaching control. Where uncertainties exist, using:

$$V = \frac{1}{2} S^2 \tag{4}$$

as a Lyapunov function candidate for S, the reaching control is selected as follows:

$$u_{reching} = -KSgn(S) \quad (5)$$

where K is the reaching term. A sufficient condition to guarantee the finite time attractiveness of $S = 0$ is to ensure finite time stability condition as:

$$\dot{V} = S\dot{S} \le -\eta|S| \quad (6)$$

where η is a strictly positive constant, which implies that:

$$t_{reaching} \le \frac{|S(0)|}{\eta} \quad (7)$$

In order to satisfy sliding condition (6) despite uncertainty on the dynamics system (1), yield

$$K = \alpha + \eta \quad (8)$$

Where α denotes the maximum system uncertainty.

Finally, SM controller is designed as follows:

$$u = u_{eq} - (\alpha + \eta)Sgn(S) \quad (9)$$

The SM controller contains the discontinuous nonlinear function $Sgn(S)$. This function in the control input can cause chattering problem. One approach to eliminate chattering is to use a continuous approximation of the discontinuous SM controller [30-32]. In this paper, we approximate $Sgn(S)$ function by $Tanh(\beta S)$ function. Hence, the approximation of SM controller is as:

$$u = u_{eq} - (\alpha + \eta)Tanh(\beta S) \quad (10)$$

The value of parameter β in this function is for adjusting the boundary layer width.

Sliding Phase

The type of stability in this phase is dependent on the type of combination of the system states and introduction of the sliding variable. In conventional SM control scheme, linear sliding variable has been introduced as:

$$S = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{X} \quad (11)$$

where λ is a strictly positive constant, $\tilde{X} = X - X_d$ and X_d is desired state vector. Assume that controller must be designed in a waythat the system state reaches thedesired state. The tracking problem for $X = X_d$ is equivalent to making $S = 0$ [31]. For example, consider a nonlinear normal system with relative degree 2 as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x_1, x_2) + g(x_1, x_2)u + w \qquad, |w| \le \alpha \end{aligned} \quad (12)$$

For stabilization with conventional SMC, the linear sliding variable according to (11) is introduced as:

$$S = x_2 + \lambda x_1 \quad (13)$$

The dynamic behavior of (13) on the sliding surface is

$$S = \dot{x}_1 + \lambda x_1 = x_2 + \lambda x_1 = 0 \quad (14)$$

In the phase plane (x1–x2 plane), S=0 represents a line, called sliding surface, passing through the origin with a slope equal to $-\lambda$. In the sliding mode (S=0) we have

$$\dot{x}_1 = -\lambda x_1 \quad (15)$$

Then, the first state variable is expressed as

$$x_1(t) = x_1(0)e^{-\lambda t} \quad (16)$$

It should be noted that λ must be a positive real constant for achieving system stability. As shown in (16), the state variable x_1 is asymptotically or exponentially stable. For finite time stabilizing the state variables, TSM control is designed byintroducing nonlinear sliding variable as follows [27]:

$$S = x_2 + \lambda x_1^{p/q} \quad (17)$$

where $\lambda > 0$ is a design constant, both p and q are positive odd integers and satisfy the following condition [33]:

$$1 < p/q < 2 \quad (18)$$

Another equivalent form of the TSM manifold can be expressed as follows [27]:

$$S = x_2 + \lambda |x_1|^{\gamma} Sgn(x_1) \quad (19)$$

where $0 < \gamma < 1$.

When the sliding variable (19) reaches thesliding surface S=0, motion of system can be described by the following nonlinear differential equation:

$$S = x_2 + \lambda x_1^{p/q} = 0 \quad (20)$$

that is:

$$\dot{x}_1 = -\lambda x_1^{p/q} \quad (21)$$

If the control u is designed to make system (14) satisfy the existence condition of the sliding-mode (11), the system can reach S = 0 within finite-time. Suppose that $t_r$is the time when S reaches zero from an initial condition $S(0) \ne 0$, that is, $S(t) = 0, \forall t \ge t_r$. From equations (20) and (21), it can be seen that if the system states (x1, x2) reach the sliding surface S = 0, then they will stay on S = 0 and converge to zero within finite-time. The system states in the ideal sliding-mode can be expressed [27, 33]:

$$x_1(t) = \begin{cases} \left(x_1^{1-q/p}(t_r) - \frac{\lambda(p-q)}{p} t \right)^{\frac{p}{p-q}} \operatorname{sgn} x_1(t_r) & t < t_s \\ 0 \quad t \ge t_s \end{cases} \quad (22)$$

$$x_2(t) = -\lambda x_1^{q/p}(t)$$

wheret_s (sliding time) can be calculated as follows:

$$t_s = \frac{p}{\lambda(p-q)} x_1^{1-q/p}(t_r) \quad (23)$$

And $t_r$is the time when S reaches zero from an initial condition.

In order to eliminate the chattering in the SMC, a saturation function sat(·) is generally used to replace the signum function sgn(·). The relationship between the steady-state errors of the SMC system and the width of the layer ϕ surrounding the sliding-mode manifold S = 0 is given by

$$\begin{cases} |x_1(t)| < \left(\dfrac{\phi}{\lambda}\right)^{p/q} & \text{subject to } |S| < \phi \\ |x_2(t)| < 2\phi \end{cases} \tag{24}$$

Note that there are three parameters which need to be designed for TSM manifold (19), p, q and λ (or two parameters γ and λ for TSM manifold (21)). These parameters can be determined based on the requirements ofthe fastness (23) or the steady-state tracking precision (24) of the sliding-mode system.

Control u in system (14) can be designed as follows:

$$u = -g^{-1}(x)\left(f(x) + \lambda\frac{p}{q}x_1^{q/p-1}x_2 + (\alpha_w + \eta)\,\mathrm{sgn}(S)\right) \tag{25}$$

where α_w is the bound of uncertainty. It can be seen in the TSM control (25) that the second term containing $x_1^{q/p-1}x_2$ may cause a singularity to occur if $x_2 \neq 0$, when $x_1 = 0$. The singularity problem may occur in the reaching phase when there is insufficient control to ensure that $x_2 \neq 0$, when $x_1 = 0$.

In order to overcome the singularity problem in the conventional TSM systems, several methods have been proposed. For example, one approach is to switch SM between TSM and linear hyperplane-based SM. Another approach is to transfer the trajectory to a pre-specified open region where TSM control is not singular. These methods adopt indirect approaches to avoid the singularity. In this paper, a simple NTSM is used, which is able to avoid this problem completely. The NTSM model is described as follows [27, 33]:

$$S = x_1 + \lambda x_2^{\,p/q} \tag{26}$$

For this sliding variable, the nonsingular TSM is designed as:

$$u = -g^{-1}(x)\left(f(x) + \lambda\frac{p}{q}x_2^{2-q/p} + (\alpha_w + \eta\frac{p}{\lambda q}|x_2|^{1-q/p})Sgn(S)\right) \tag{27}$$

It should be noted that the NTSM control (27) is always nonsingular in the sliding phase [27, 33].

Guidance Law Design

In this section, first, the model of guidance loop dynamical system is introduced and then, NTSM guidance law for intercepting the desired impact angle is designed.

Modeling Guidance Loop

In this section, a model for guidance loop is formulated. Consider a two-dimensional interceptor and target engagement as shown in Fig. 1.

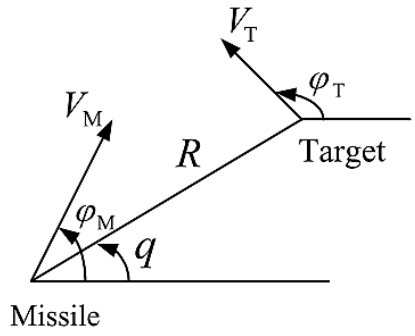


Fig. 1. Missile-target engagement geometry

It is assumed that the missile and the target are point masses moving in plane. Then, the missile-target engagement model shown in Fig. 1 can be described by the following nonlinear differential equations [34]:

$$\frac{d}{dt}(R) = \dot{R} = V_T\cos(q - \phi_T) - V_M\cos(q - \phi_M) \tag{28}$$

$$\frac{d}{dt}(q) = \dot{q} = -V_T\sin(q - \phi_T) + V_M\sin(q - \phi_M) \tag{29}$$

$$\frac{d}{dt}(\phi_M) = \dot{\phi}_M = \frac{1}{V_M}A_M \tag{30}$$

$$\frac{d}{dt}(\phi_T) = \dot{\phi}_T = \frac{1}{V_T}A_T \tag{31}$$

where R is the relative range between target and interceptor q is the LOS angle with respect to a reference axis; $\dot{q}$ is the line of sight rate; V_M and V_T represent the interceptor and target velocities, respectively; ϕ_M and ϕ_T represent the flight path angles of the target and missile, respectively; and A_M and A_T represent the interceptor and target accelerations, respectively.

Differentiating Eqs. (28) and (29) with respect to time yields [3, 34]:

$$\ddot{R} = R\dot{\sigma}^2 - A_{M,R} + A_{T,R} \tag{32}$$

$$\ddot{q} = \frac{1}{R}\left(-2\dot{R}\dot{\sigma} - A_{M,q} + A_{T,q}\right) \tag{33}$$

where, $A_{T,R}$ and $A_{M,R}$ denote the accelerations of the target and missile along the LOS, respectively; and $A_{T,q}$ and $A_{M,q}$ denote the accelerations of the target and missile normal to the LOS, respectively [30]. By considering state variables as $[x_1\ x_2\ x_3\ x_4] = [R\ \dot{R}\ q\ \dot{q}]$, control input $[u_1\ u_2] = [A_{M,R}\ A_{M,q}]$, outputs $[y_1\ y_2] = [R\ q]$ and $[w_1\ w_2] = [A_{T,R}\ A_{T,q}]$ as uncertainty, the state equations are rewritten as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1 x_4^2 - u_1 + w_1 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -\frac{2x_2x_4}{x_1} - \frac{1}{x_1}u_2 + \frac{1}{x_1}w_2 \end{aligned} \tag{34}$$

The first object of guidance law is intercepting thetarget with desired angle ($x_3 \to x_{3d}$) and another object is to nullify the LOS rate ($x_4 \to 0$) in finite time. For these objectives in the next section,NTSM control is used for designing u_2 . Also, assume that the interceptor has no accelerations along LOS and $A_{M,R} = 0$.

Impact Angle NTSM Guidance Law

In this section, the design procedure of NTSM guidance law is presented for the system equations given by (34). For these, propose we start with introducing nonsingular terminal sliding variable as follows:

$$S = x_3 + \lambda(x_4 - x_{4d})^{p/q} \tag{35}$$

Therefore, the sliding dynamics is achieved:

$$\begin{aligned} \dot{S} &= \dot{x}_3 + \lambda\frac{p}{q}x_4^{\,p/q-1}\dot{x}_4 = \\ & x_4 + \lambda\frac{p}{q}x_4^{\,p/q-1}\left(-\frac{x_2x_4}{x_1} - \frac{1}{x_1}u_2 + \frac{1}{x_1}w_2\right) \end{aligned} \tag{36}$$

Now, taking the controller u_2 in (38) as follows:

$$u_2 = u_{eq} - KSgn(S) \tag{37}$$

where u_{eq} is chosen to cancel the known terms on the right-hand side of (36), and (9) is considered as a Lyapunov function candidate for S .

Then, the reaching control is selected as (10) and a sufficient condition to guarantee the finite time attractiveness of $S = 0$ for $S \neq 0$ is to ensure (11), which implies that (12) is reaching time, controllers is given as

$$\begin{aligned} u_2 &= -2x_2x_4 + \lambda\frac{p}{q}x_4^{2-p/q}x_3 + \\ & (\alpha_{w_2} + \eta\frac{p}{\lambda q}|x_4|^{1-q/p}x_1)Sgn\left((x_3 - x_{3d}) + \lambda x_4^{\,p/q}\right) \end{aligned} \tag{38}$$

Thus, the missile acceleration component in LOS coordinate is given as

$$\begin{aligned} A_{Mq} &= -2\dot{R}\dot{q} + \lambda\frac{p}{q}\dot{q}^{2-p/q}R + \\ & (\alpha_{w_2} + \eta\frac{p}{\lambda q}|\dot{q}|^{1-q/p}R)Sgn\left((q - q_d) + \lambda\dot{q}^{\,p/q}\right) \end{aligned} \tag{39}$$

This missile acceleration in LOS coordinate or its continuous approximation globally stabilizes the $(q,\dot{q}) = (q_d, 0)$ in finite time without singularity in guidance command.

Numerical Simulation

Numerical simulations are performed to investigate the performance of the proposed guidance law. In this section, we consider asituation in which the initial relative distance is 5 km, the initial closing velocity is 300 m/s ($V_m = [250,0]\,m/s$, $V_t = [50,0]\,m/s$), and the initial values of LOS angle and LOS rate are zero.

The proposed nonsingular TSMG is compared with conventional SM guidance which is designed byintroducing conventional sliding variable and TSM guidance with (19) as sliding variable and (27) as control input. The lateral acceleration using terminal SMC is designed as follows [26]:

$$\begin{aligned} A_{Mq} &= -2\dot{R}\dot{q} + \lambda R\frac{p}{q}q^{\,p/q-1}\dot{q} + \\ & (\alpha_{w_2} + \eta R)Sgn\left(\dot{q} + \lambda(q - q_d)^{p/q}\right) \end{aligned} \tag{40}$$

In all guidance laws, for implementation reasons and for removing chattering, the $Sgn(S)$ function is replaced with $Tanh(\beta S)$ function. The value of parameter β in this function isadjusting the boundary layer width.

These guidance laws are compared in 3 different scenarios. In the first scenario, the desired LOS angle (σ_d) is 30 degrees and target has no maneuver.In the second scenario, the desired LOS angle (σ_d) is 30 degrees and target has 10 m/s2 acceleration. In the third, the desired LOS angle (σ_d) is -10 degrees and target has no maneuver that leads to singularity. The other parameter values are listed in Table 1.

Table 1. The value of parameters in all three scenarios

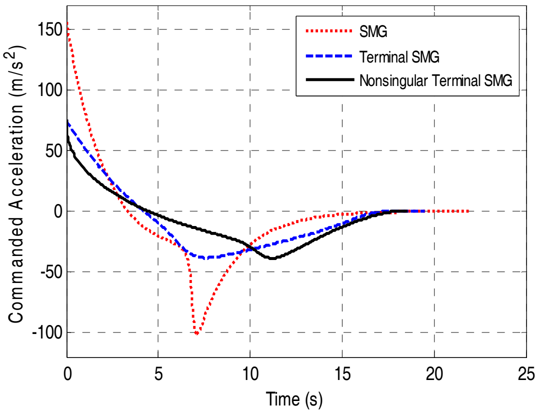
Guidance Law	λ	η	β	p/q
SMG	0.4	0.02	500	-
TSMG	0.1	0.015	130	2/5
NTSMG	20	0.01	200	5/3

Scenario I

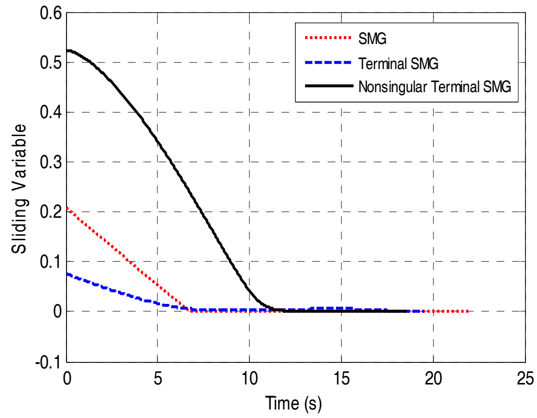
In this scenario, the performance of guidance laws for intercepting non-maneuvering targets with desired angle is checked. Therefore, desired LOS angle (σ_d) is 30 degrees and target acceleration is zero. In this case,Fig. 2 shows that the peak of commanded acceleration (control signal) using proposed nonsingular TSMG is lower than other guidance laws. Also, in all three guidance laws, sliding variables reach zero in desired time that is achieved from equation (12).

LOS angle and its angular rate are plotted in figures 3-(a) and 3-(b). As seen in figures, the LOS angle (x_3) reaches desired value ($\sigma_d = 30\deg$) before interception target. Also, LOS rate reaches zero using all three guidance laws. Note that, these variables using conventional SMG are exponential stable in sliding phase, but using TSM and proposed nonsingular TSM guidance laws, these variables are finite time stable in both reaching and sliding phases. Figure 4-(a) shows the linear sliding surface in conventional SMG and nonlinear sliding surfaces in two other guidance laws that guarantee finite time stability in sliding phase.

From Fig. 4-(b), it is clear that the missile interceptstarget with desired angle. As shown in this figure, first the altitude of missile is increased and LOS angle reaches desired value. Then, missile continueswith this angle and interceptsthe target. It is clear from Fig. 5 and Table 2, that the velocity losses, interception time, control effort and the magnitude of commanded acceleration in the proposed guidance law are shorter than two other guidance laws.

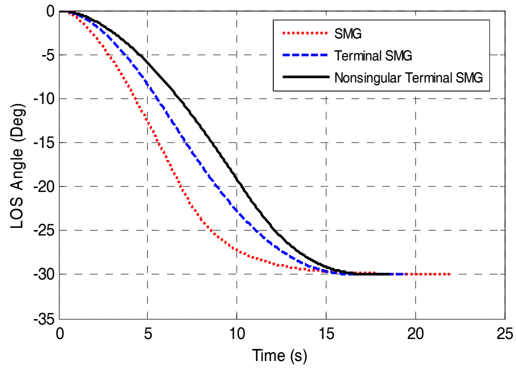


(a)

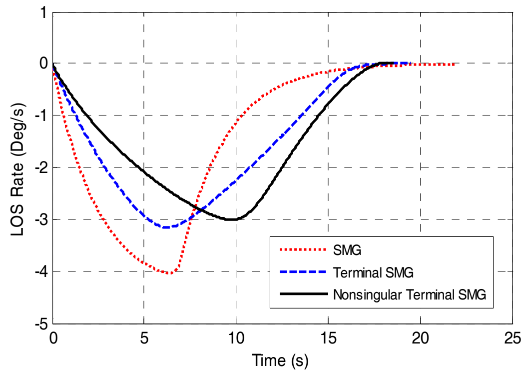


(b)

Fig 2. (a) Commanded Acceleration (b) Sliding Variable

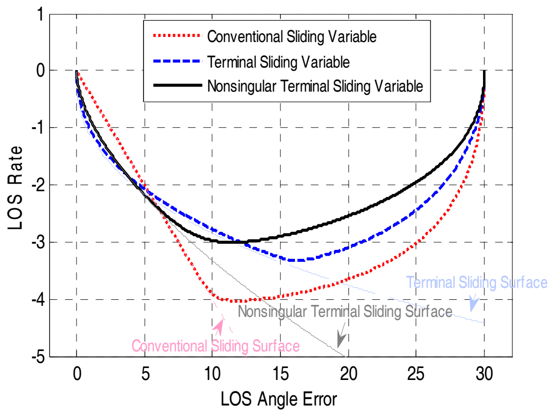


(a)

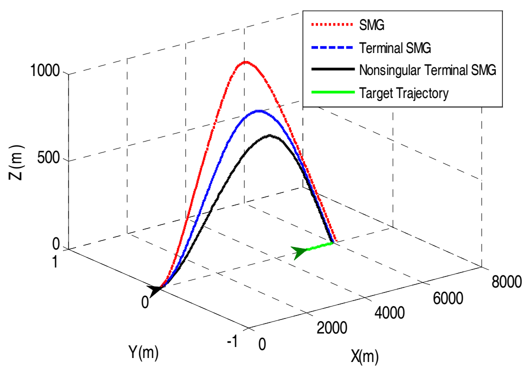


(b)

Fig 3. (a) Line of Sight angle (b) Line of Sight rate



(a)



(b)

Fig 4. (a) Phase Plane (b) Interception Plan

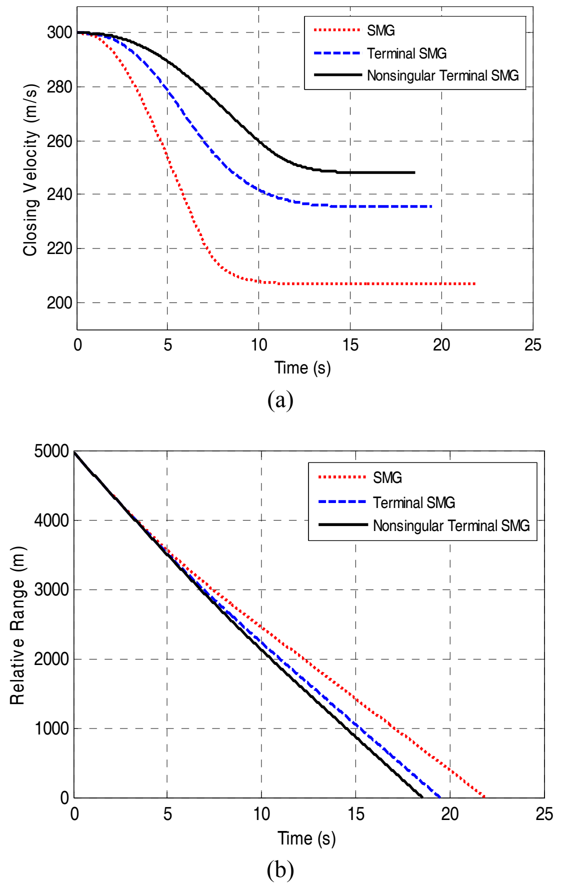


Fig 5. (a) Closing Velocity (b) Relative Range

Table 2. The Information for Scenario I

Guidance Law	Interception Time (s)	Energy (m2/s3)	Peak of Acceleration (m/s2)
SMG	22	34550	155
TSMG	19.5	14500	75
NTSMG	18.75	9400	70

Scenario II

In this scenario, the performance of guidance laws for intercepting the maneuvering targets with desired angle is checked. Therefore, target acceleration is 10 m/s2. In this case,Fig. 6 shows that the peak of commanded acceleration using proposed NTSMG is lower than other laws and sliding variables reachzero in desired time. As seen in Fig. 7, the LOS angle reaches desired value before interception into target and LOS rate reaches zero. As shown in Fig. 8, LOS angle reaches desired value and then, missile continues with this angle and interceptsthe target. It is clear from Fig. 9 and Table 3, that the velocity losses, interception time, control effort and the magnitude of commanded acceleration in proposed guidance law areshorter than other guidance laws.

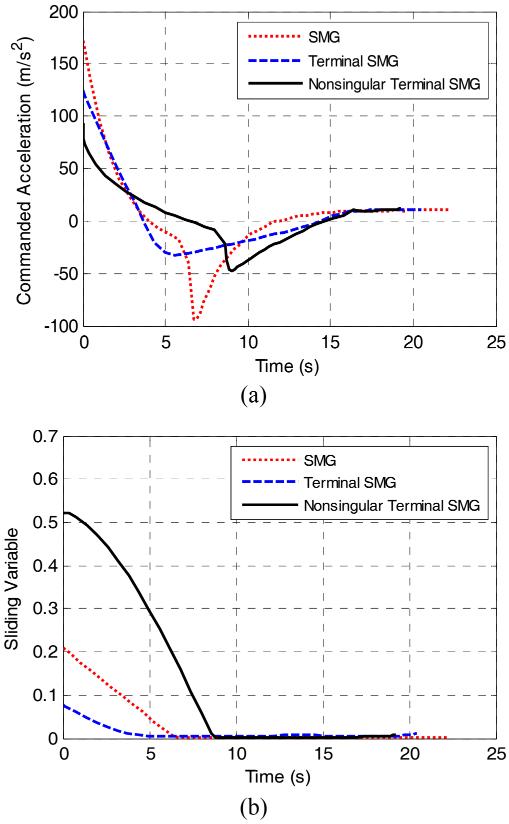


Fig 6. (a) Commanded Acceleration (b) Sliding Variable

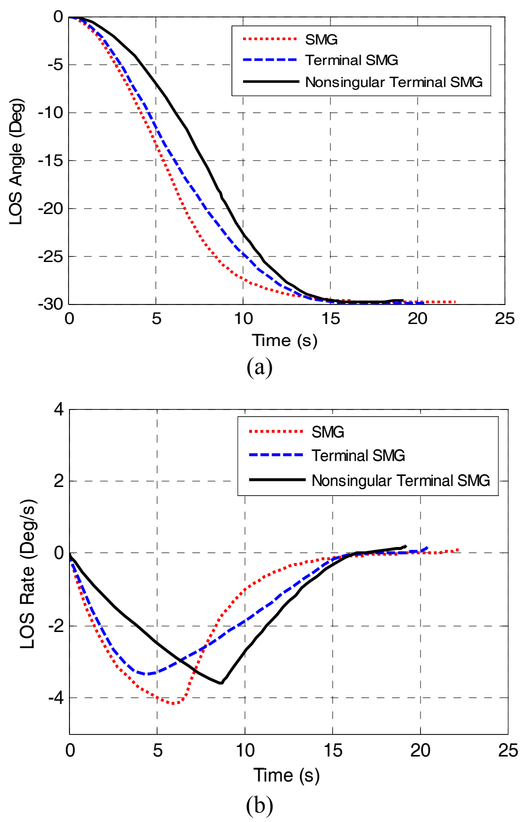
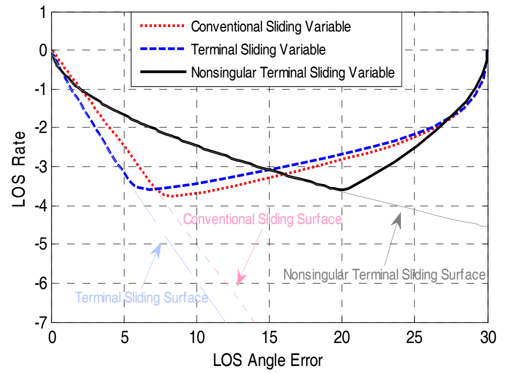
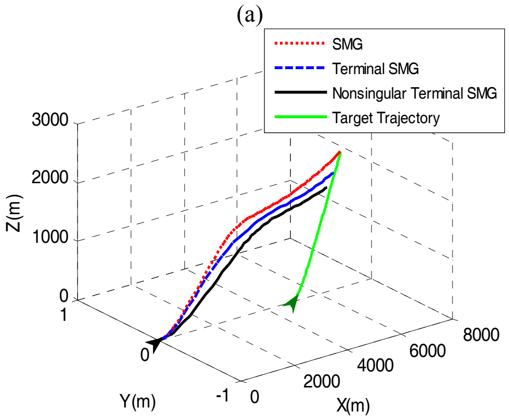


Fig 7. (a) Line of Sight angle (b) Line of Sight rate

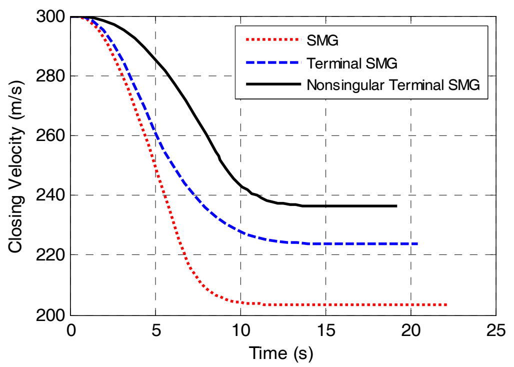


(a)

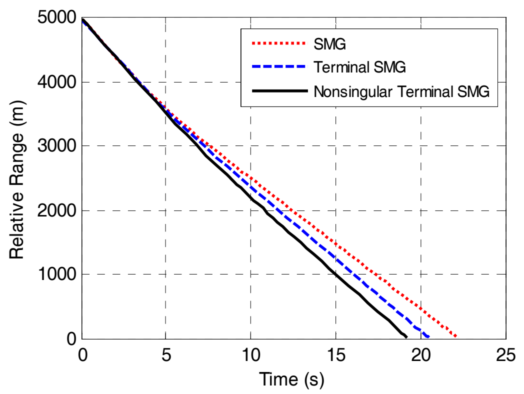


(b)

Fig. 8. (a) Phase Plane (b) Interception Plan



(a)



(b)

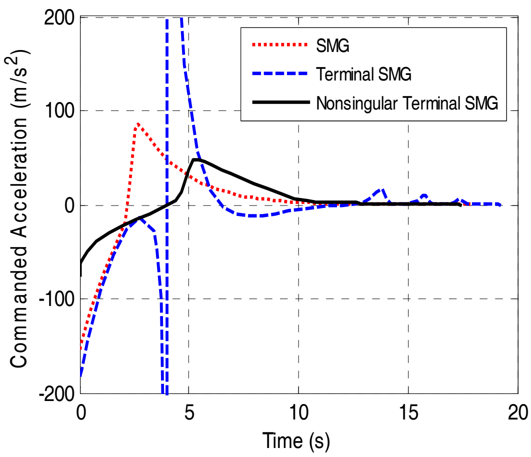
Fig. 9. (a) Closing Velocity (b) Relative Range

Table 3. The Information for Scenario II

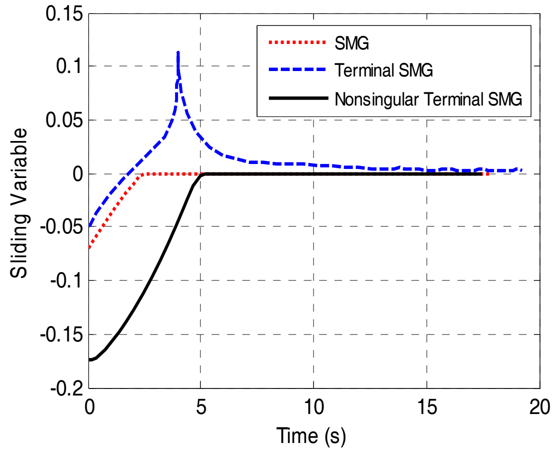
Guidance Law	Interception Time(s)	Energy (m^2/s^3)	Peak of Acceleration (m/s^2)
SMG	22.2	37000	170
TSMG	20.5	23500	125
NTSMG	19.25	12200	94

Scenario II

In this scenario, the performance of guidance laws is checked when singularity occurrs. Therefore, desired LOS angle (σ_d) is -10 degrees and target acceleration is zero. In this case, Fig.s 10 and 11 show that the singularity occurs in TSMG law when LOS angle reaches desired angle and LOS rate has non-zero value. This acceleration command is not implementable and leads to singularity in LOS rate, too. It is clear from Fig. 13 and Table 4, that this singularity leads to an increase in the velocity losses, interception time, and control effort.

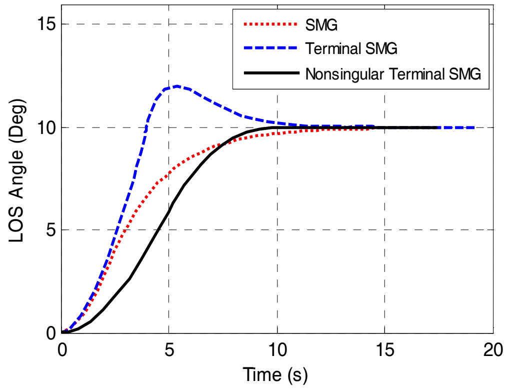


(a)

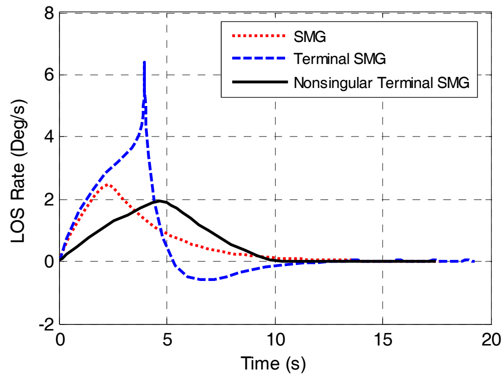


(b)

Fig 10. (a) Commanded Acceleration (b) Sliding Variable

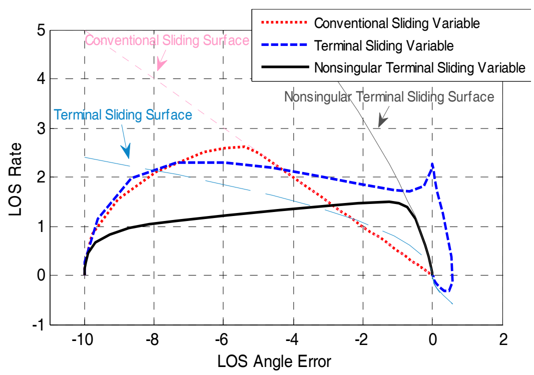


(a)

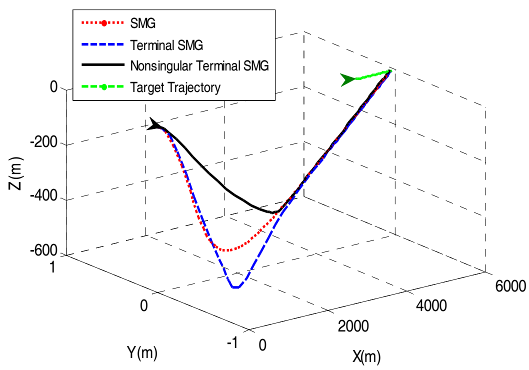


(b)

Fig. 11. (a) Line of Sight angle (b) Line of Sight rate

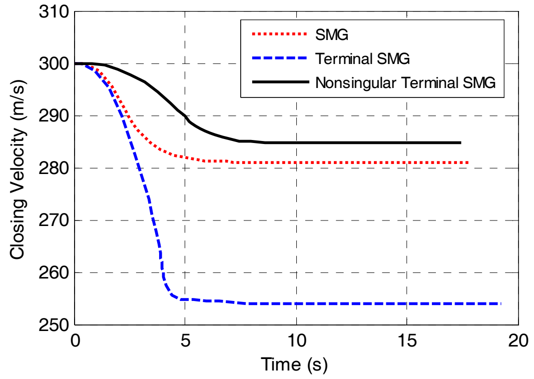


(a)

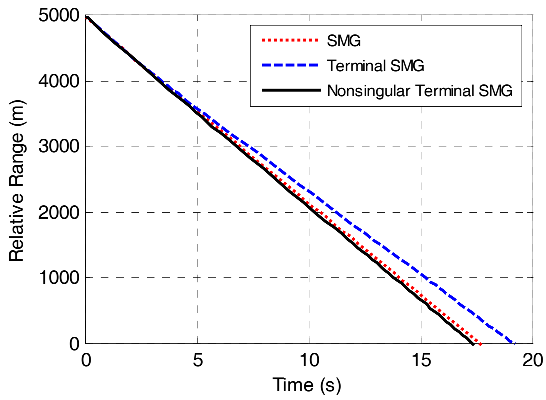


(b)

Fig. 12. (a) Phase Plane (b) Interception Plan



(a)



(b)

Fig. 13. (a) Closing Velocity (b) Relative Range

Table 4. The Information for Scenario III

Guidance Law	Interception Time(s)	Energy (m2/s3)	Peak of Acceleration (m/s2)
SMG	17.8	27200	155
TSMG	19.23	5314000	500000
NTSMG	17.4	8250	78

Conclusion

The new guidance law which isproposed here can adjust LOS angle to the predefined LOS angle. The structure of the proposed guidance law is developed based on NTSM control theory. This method guarantees the convergence of LOS angle and rate without singularity in commanded acceleration as control signal. The proposed guidance law has simple structure and is robust in the pretense of uncertainties such as target maneuver. Simulation results show the effectiveness and robustness of the proposed guidance law in comparison with the conventional and terminal SMG.

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