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Lyapunov-basedAdaptive SmoothSecond-order Sliding Mode Guidance Law with Proving Finite Time Stability

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A new adaptive smooth second-order sliding mode control is proposed for uncertain nonlinear systems in this paper. The finite time stability is proved using a Lyapunov technic. The proposed controller consists of an adaptive term equal to the uncertainty in finite time. This algorithm is used to design terminal guidance law for homing interceptors to intercept maneuvering targets. The guidance law generates smooth acceleration commands and the control signal is able to stabilize relative lateral velocity in a desired time. Finally, the proposed guidance law is compared with the second-order sliding mode guidance law from carried out simulations.

Keywords: Second-order sliding mode, Finite time convergence, Guidance law, Maneuvering target

Nomenclature

s	Sliding Variable
u	Control Input Signal
$d(t)$	Uncertainty
k	Reaching Term
t_r	Reaching Time
V	Lyapunov function
L_d	Bound of the Uncertainty
q	Line of Sight Angle
$\dot{q}$	Line of Sight Rate
R	Relative Range
$\dot{R}$	Closing Velocity
A_M	Missile Lateral Acceleration
V_M	Missile Velocity
V_T	Target Velocity
ϕ_M	Missile Flight path angle
ϕ_T	Target Flight path angles

Introduction

Sliding mode (SM) control is a very popular strategy to control uncertain nonlinear systems such as guidance loop. Its main features are the robustness of closed-loop system and the finitetime convergence. In the classical first orderSM control, for the sliding variable to be stable, a switching function has to be used in the control law, which causes chattering of the control signals. Chattering effects in conventional SM controllers is one of the main disadvantages for practical applications[1-5]. In [6-8], chattering effect analysis is performed. Due to chattering, the application of the conventional SM control theory in the outer loop of a multi-loop control system such as guidance loop is not possible. This is because the commands generated by the guidance law cannot be followed by the autopilot in the inner control loop [9].

An approach to remove chattering effects is to replace the discontinuous switching function by a continuous approximated function. However, this method leads to precision reduction [5, 10 and 11]. An alternative method is to use the high order sliding mode (HOSM) control law. However, the main challenge of HOSM control is the use of high order time derivatives of the sliding variable. The only exception is the super twisting (ST) controller [10, 12 and 13]. Chattering effect is not eliminated, but attenuated in the ST controller due to a discontinuous function present under the integral. Therefore, ST is not very smooth [9]. In the recent years, time estimation for SOSM controllers via Lyapunov function designing has become an attractive research area [14]. In SOSM algorithms, the geometrical finite-time convergence analysis has been

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done without the use of Lyapunov function approach. Some approaches depend on the homogeneity principle [15] which does not provide estimation for the convergence time.A homogeneity-based SOSM guidance law presented in [9] is only accurate for particular nonlinear systems. Then, a disturbance cancellation term is estimated using an observer.

For the adaptation of uncertainty, adaptive SM controllers have been proposed [31-33] with the interest being the adaptation of uncertainty effects. Then, a reduced gain induces lower chattering [10, 19]. Some various applications of adaptive SM control are proposed in [10, 19-22].

Sliding mode control theories have been applied to many guidance problems. Using sliding (SM) control theory, a nonlinear and robust guidance law against maneuvering targets can be designed. The classical first-order SM and its continuous approximation versionsare used for designing simple guidance laws in [23-30]. By using SOSM control, guidance law is presented in [9]. This guidance law is based on uncertainty observer and the other problem with this method is homogeneity-based finite time stability proof. Also, observer-based SM guidance laws are designed in [16-18]. The main drawback in these guidance algorithms is the absence of a formal closed-loop system stability proof.

This paper modifies the smooth second-order sliding mode (SSOSM) control proposed in [9]. In the proposed adaptive Lyapunov-based SOSM control algorithm, homogeneity-based SSOSM control is compared with when:

1) The uncertainty is estimated with an adaptive term and canceled in feedback;therefore, disturbance observer is not used.

2) The finite time stability of the closed-loop system is proved based on the Lyapunov theorem; therefore, the calculation of the convergence time is available.

The proposed control algorithm is used for guidance law design. This guidance law generates a smooth acceleration command and ensures finite time convergences of the relative lateral velocity and the estimation of target acceleration normal to line of sight.

Adaptive Smooth Second-order Sliding Mode Control Law

Consider relationship (1) as the sliding dynamics introduced in a nonlinear system.

$$\dot{s} = d(t) + u \tag{1}$$

The problem is to design controller u that drives the sliding variable s to zero in a finite time in the presence of the uncertainty $d(t)$. Assume $d(t)$ is the uncertain term with $|d(t)| \le L_d$ and its first derivative is $g(t)$ with $|g(t)| \le L_g$.

The conventional first-order SM controller is able to stabilize the sliding variable by control input

$$u = -k\,sign(s) \tag{2}$$

where k is the reaching term. Considering $V = \frac{1}{2}s^2$ as a Lyapunov function, to meet the requirements, finite time convergence of s must be

$$\dot{V} = s\dot{s} \le -\eta|s| \tag{3}$$

where η is a positive constant, which implies reaching time is $t_r \le |s(0)|/\eta$ [5].

The conventional SM controller (2) contains the discontinuous nonlinear function $sign(s)$ with gain $k = L_d + \eta$. This function can cause chattering effects.

Based on the continuous approximation method in boundary layer, the controller (2) can be replaced with $u = -k\,sat(s)$. This method brings a finite steady state error.

The control law using second-order super twisting algorithm is obtained [9]

$$\begin{cases} u = -k_1|s|^{1/2}\,sign(s) + w \\ \dot{w} = -k_2\,sign(s) \end{cases} \tag{4}$$

However, this control law is not completely smooth [9]. Also, a smooth second-order sliding mode (SSOSM) control is given below

$$\begin{cases} u = -z_1 - k_1|s|^{m/(m+1)}\,sign(s) + w \\ \dot{w} = -k_2|s|^{(m-1)/(m+1)}\,sign(s) \end{cases} \tag{5}$$

With $k_1, k_2 > 0$ and $m \ge 1$ presented in [9] for system (1) with existence estimation of the uncertainty z_1. The stability of this algorithm is based on homogeneity technic and is not based on Lyapunov method. Therefore, reaching time estimation is not available using controller (5).

In this paper, a modified version of controller (5) is proposed in the following theorem.

Theorem 1: The adaptive smooth second-order sliding mode controller (6) with conditions (7) ensures the convergence of $s \to 0$ and $(d(t) + w) \to 0$ in a finite time.Parameters k_1, k_2, k_3 and η are positive constants and L_d and L_g are bounds of the uncertainty and its first derivative.

$$\begin{cases} u=-k_1|s|^{\alpha} sign(s)+w-k_3\xi \\ \dot{w}=-k_2|s|^{\beta} sign(s) \\ \dot{\xi}=k_2k_3|\xi||s|^{\beta} sign(s)-k_4 sign(\xi) \end{cases} ,0<\alpha<1 \quad ,0<\beta<1 \tag{6}$$

$$\begin{cases} k_1,k_2>0 \\ k_4=L_g\left(L_d+|w|\right)+\eta \\ \eta = \text{positive constant} \end{cases} \tag{7}$$

Proof: Using controller (6) in the uncertain nonlinear system (1), we have closed-loop dynamics:

$$\begin{cases} \dot{s}=-k_1|s|^{\alpha} sign(s)-k_3\xi+d+w \\ \dot{w}=-k_2|s|^{\beta} sign(s) \\ \dot{\xi}=k_2k_3|\xi||s|^{\beta} sign(s)-k_4 sign(\xi) \end{cases} \tag{8}$$

and by introducing the estimation error as $e=d(t)+w$, the error dynamics in closed-loop system (8) can be written as

$$\begin{cases} \dot{s}=-k_1|s|^{\alpha} sign(s)-k_3\xi+e \\ \dot{e}=g(t)-k_2|s|^{\beta} sign(s) \\ \dot{\xi}=k_2k_3|\xi||s|^{\beta} sign(s)-k_4 sign(\xi) \end{cases} \tag{9}$$

Now, consider the following Lyapunov function:

$$V=\frac{k_2}{1+\beta}|s|^{1+\beta}+\frac{1}{2}e^2+|\xi| \tag{10}$$

which is positive definite with $k_2>0$.

By taking the time derivative of the Lyapunov function (10), the following is obtained:

$$\begin{aligned} \dot{V}&=k_2|s|^{\beta}\frac{s}{|s|}\dot{s}+e\dot{e}+\frac{\xi}{|\xi|}\dot{\xi}=k_2|s|^{\beta}\frac{s}{|s|}\left(-k_1|s|^{\alpha}sign(s)-k_3\xi+e\right) \\ &+e\left(g(t)-k_2|s|^{\beta}sign(s)\right)+\frac{\xi}{|\xi|}\left(k_2k_3|\xi||s|^{\beta}sign(s)-k_4 sign(\xi)\right) \\ &=-k_1k_2|s|^{\alpha+\beta}-k_2|s|^{\beta}\frac{s}{|s|}k_3\xi+k_2|s|^{\beta}\frac{s}{|s|}e+g(t)e \\ &-k_2|s|^{\beta}sign(s)e+k_2k_3\xi|s|^{\beta}sign(s)-k_4= \\ &-k_1k_2|s|^{\alpha+\beta}+g(t)e-k_4 \end{aligned} \tag{11}$$

Choosing $k_1,k_2>0$ and $k_4=L_g\left(L_d+L_w\right)+\eta$, where η is a strictly positive constant, yields:

$$\dot{V}=-k_1k_2|s|^{\alpha+\beta}+g(t)e-L_g\left(L_d+L_w\right)-\eta\le-\eta \tag{12}$$

The condition

$$\dot{V}\left(x(t)\right)\le-\eta \tag{13}$$

that implies

$$0\le V\left(x(t)\right)=V\left(x(0)\right)-\eta t \tag{14}$$

defines

$$t\le\frac{V\left(x(0)\right)}{\eta} \tag{15}$$

Therefore, condition (13) is a finite time condition [31, 32]. Hence, the closed-loop system error converges in t that is the time $V\left(x(t)\right)$ reaches zero from an initial condition $V\left(x(0)\right)$. The finite time stability of the closed-loop system error (9) is ensured.

Guidance Law Design

Consider two-dimensional interceptor-target engagement geometries as shown in Fig 1.

The missile-target engagement model shown in Fig. 1 can be described by nonlinear differential equations [9]:

$$\begin{aligned} &\dot{R}=V_R \\ &\dot{V}_R=\frac{V_q^2}{R}+A_{T,R}-\sin(q-\phi_M)A_M \\ &\dot{q}=\frac{V_q}{R} \\ &\dot{V}_q=-\frac{V_qV_R}{R}+A_{T,q}-\cos(q-\phi_M)A_M \end{aligned} \tag{16}$$

where R is the relative distance between interceptor and the target, q is named the line of sight (LOS) angle, $\dot{q}$ is the line of sight angular rate, $V_q=R\dot{q}$ is the relative lateral velocity and ϕ_M and ϕ_T are named as the flight path angles of the target and missile, respectively, $A_{T,R}$, $A_{T,q}$ are the target accelerations along and orthogonal to LOS and A_M is the interceptor acceleration. The object nullifies the relative lateral velocity ($V_q=R\dot{q}\to 0$) in a finite time that causes interception with the target.

To design guidance law using the proposed algorithm for the system equations given by (16), the sliding variable is introduced firstly, as follows:

$$s=V_q \tag{17}$$

Therefore, the sliding dynamic is identified as

$$\dot{s}=\dot{V}_q=-\frac{V_qV_R}{R}+A_{T,q}-\cos(q-\phi_M)A_M \tag{18}$$

Then, the proposed smooth second-order sliding mode guidance law is achieved

$$\begin{cases} A_M = \dfrac{1}{\cos(q-\phi_M)}\begin{pmatrix} -\dfrac{V_q V_R}{R} + k_1 \left|V_q\right|^{\alpha} sign(V_q) \\ +w - k_3\xi \end{pmatrix} \\ \dot{\xi} = k_2 k_3 \left|\xi\right| \left|V_q\right|^{\beta} sign(V_q) - k_4 sign(\xi) \\ \dot{w} = -k_2 \left|V_q\right|^{\beta} sign(V_q) \end{cases} \quad (19)$$

This guidance law stabilizes V_q and $e = A_{T,q} + w$ in a finite time without chattering effects in guidance command.

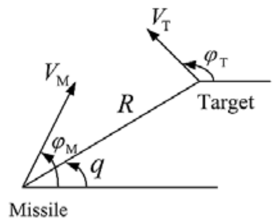

Fig. 1. Missile-Target Engagement Geometry

Numerical Simulation Results

In this section, simulations are performed to investigate the performance of the proposed guidance lawassuming the initial relative distance is 40 km and the initial missile and target velocities are $V_m = 800\, m/s$, $V_t = 700\, m/s$, respectively.

For comparison, the smooth second-order sliding mode guidance (SSOSMG) law [9]

$$A_M = \frac{1}{\cos(q-\phi_M)}\begin{pmatrix} -N'\dfrac{V_q V_R}{R} - c_0 \dfrac{V_R}{2\sqrt{R}} \\ +k_1 \left|s\right|^{2/3} sign(s) \\ +k_2 \int \left|s\right|^{1/3} sign(s) d\tau \\ +\hat{A}_{T,q} \end{pmatrix} \quad (20)$$

is also considered. $\hat{A}_{T,q} = z_1$ is the estimation of target acceleration normal to LOS achieved using the following observer

$$\begin{cases} \dot{z}_0 = v_0 - \cos(q-\phi_M)A_m - \dfrac{V_q V_R}{R} - c_0 \dfrac{V_R}{2\sqrt{R}} \\ v_0 = -2L^{1/3}\left|z_0 - s\right|^{2/3} sign(z_0 - s) + z_1 \\ \dot{z}_1 = v_1 \\ v_1 = -1.5L^{1/2}\left|z_1 - v_0\right|^{1/2} sign(z_1 - v_0) + z_2 \\ \dot{z}_2 = -1.1L sign(z_2 - v_1) \\ \hat{A}_{T,q} = z_1 \end{cases} \quad (21)$$

Guidance laws are compared in the two scenarios. In the first scenario, target has constant -25 m/s^{2}lateral accelerationand in the second, target maneuver is variable.

A. Scenario I

In this scenario, the target acceleration is constant -25 m/s^2 as shown in Fig. 2. In this case, an initial relative distance of 40 km is assumed, the initial missile and target velocities are $V_m = 800\, m/s$, $V_t = 700\, m/s$, respectively and the initial values of LOS angle is zero. The missile and target initial flight path angles are $\phi_M = 5$ deg and $\phi_T = 150$ deg, respectively and the other parameter values are listed in Table 1.

Table 1. Parameter values for the first scenario

Guidance Law	k_1	k_2	k_3	k_4	α	β	N'	c_0	L
Proposed Guidance Law	7	5	0.1	50	0.6	0.95	-	-	-
SSOSMG	1.5	0.5	-	-	-	-	4	0.1	1

In this case, the target acceleration normal to LOS ($A_{T,q}$), the adaptive term of proposed guidance law (w) and $\hat{A}_{T,q} = z_1$ in SSOSMG are plotted in Fig. 3. It is clear from this figure that the adaptive term in the proposed guidance law converges to the target lateral acceleration normal to LOS with a higher precision than SSOSMG. In Fig.4, the estimation errors in the proposed guidance law ($e = A_{T,q} + w$) and in SSOSMG ($e = A_{T,q} - z_1$) are plotted. It is evident that the estimation precision in the proposed guidance law is higher than SSOSMG. The interceptor lateral acceleration, relative lateral velocity and line of sight rate are shown in Fig.s 5-7. Both guidance laws generate smooth acceleration commands. In comparison to SSOSMG, precision is higher in the stabilization of the relative lateral velocity and LOS rate in the proposed guidance law.

Finally, in Fig. 8, the interception trajectory in the first scenario is plotted.

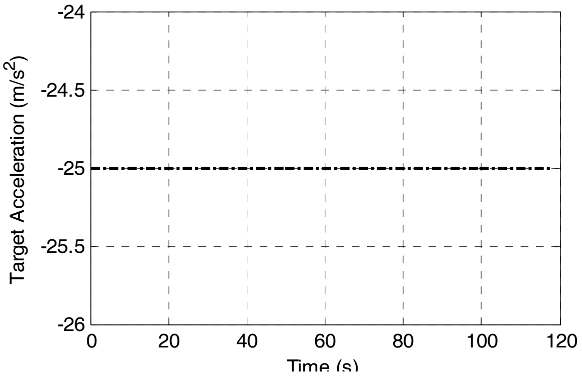

Fig. 2. Target Acceleration (A_T) in the First Scenario

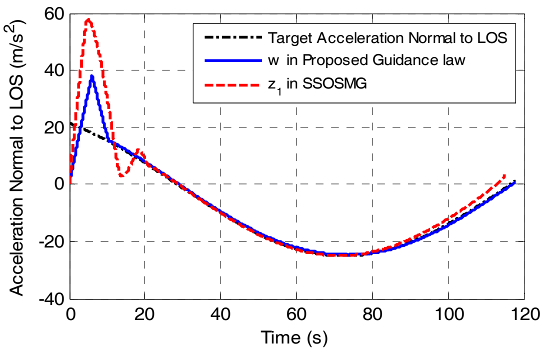

Fig. 3. Target Acceleration Normal to LOS ($A_{T,q}$), Integral Term in the Proposed Guidance Law (w) and the Estimated Term in SSOSMG (z_1) in the First Scenario

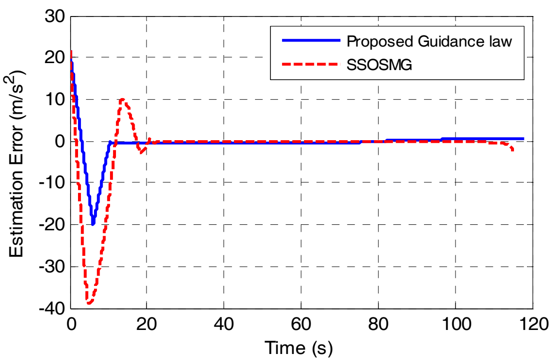

Fig. 4. Target Acceleration Estimation Error in the Proposed Guidance Law ($A_{T,q}$+w) and SOSMG ($A_{T,q}$-z_1) in the First Scenario

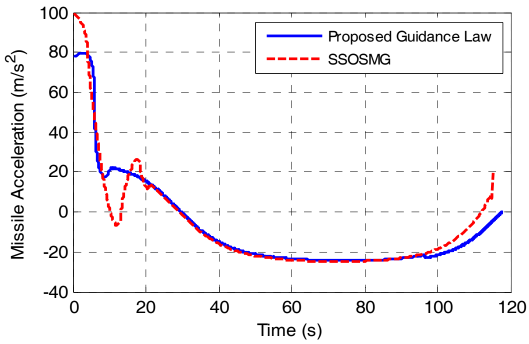

Fig. 5. Missile Lateral Acceleration ($A_{M,q}$) in the First Scenario

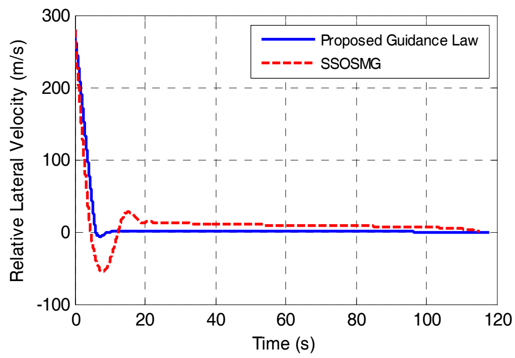

Fig. 6. Relative Lateral Velocity (V_q) in the First Scenario

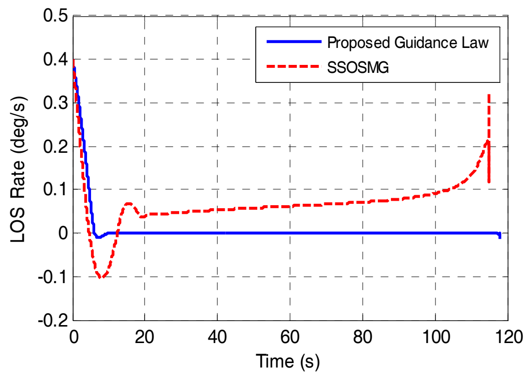

Fig. 7. Line of Sight Rate in the First Scenario

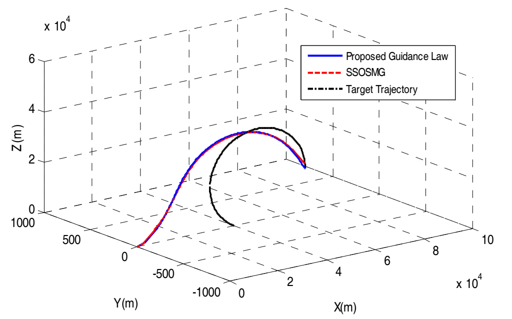

Fig. 8. Interception Trajectory in the First Scenario

A. Scenario II

In this scenario, the performance of guidance laws to intercept targets with variable maneuvers as shown in Fig.9 is addressed. In this case, initial relative distance is assumed to be 141421.4 meters, and the initial missile and target velocities are $V_m = 800\, m/s$, $V_t = 2000\, m/s$, respectively. The initial value of LOS angle is 45 degrees, the missile and target initial flight path angles are $\phi_M = 30$ deg and $\phi_T = 225$ deg, respectively and the other parameter values are listed in Table 2.

In this case, Fig.s 10 and 11 show that the adaptive term in the proposed guidance law converges the target lateral acceleration normal to LOS and the estimation error $(e = A_{T,q} + w)$ is stabilized with a higher precision than SOSMG law. As can be seen inFig. 11, the estimation error in SOSMG law is higher than the proposed law, particularly towards the end of the simulation time. As shown in Fig.s 12-14, the peak of the commanded acceleration using the proposed guidance law is lower than SSOSMG and relative lateral velocity and LOS rate converge to zero in finite time with higher precision. Finally, in Fig. 15 the interception trajectory in the second scenario is plotted.

Table 2. Parameter values for the second scenario

Guidance Law	k_1	k_2	k_3	k_4	α	β
Proposed Guidance Law	4	4	0.05	50	0.5	0.95
SSOSMG	2	0.5	-	-	-	-

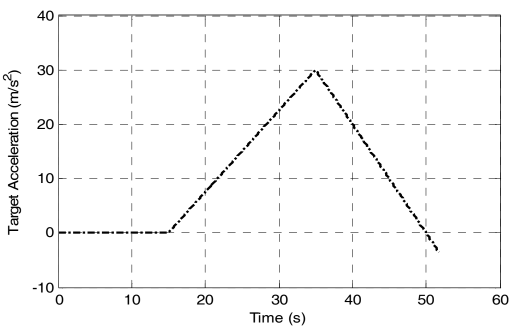

Fig. 9. The Target Acceleration (A_T) in the Second Scenario

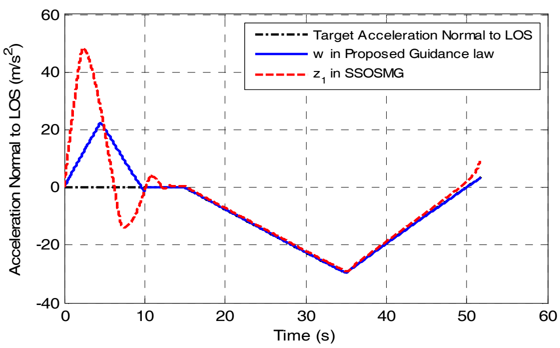

Fig. 10. Target Acceleration Normal to LOS ($A_{T,q}$), Integral Term in the Proposed Guidance Law (w) and Estimation in SSOSMG (z1) in the Second Scenario

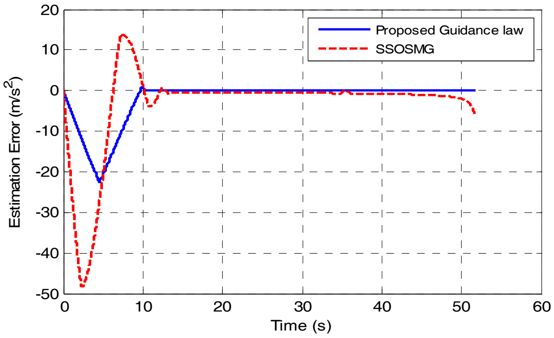

Fig. 11. Target Acceleration Estimation Error in the Proposed Guidance Law ($A_{T,q}$+w) and SOSMG ($A_{T,q}$-z_1) in the Second Scenario

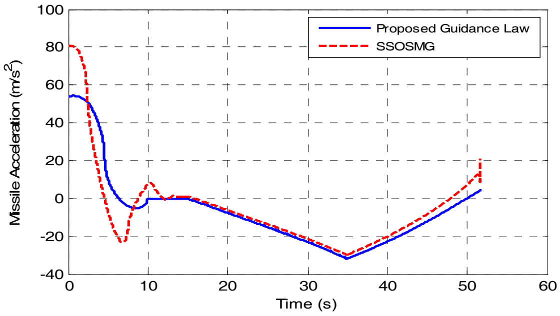

Fig. 12. Missile Lateral Acceleration ($A_{M,q}$) in the Second Scenario

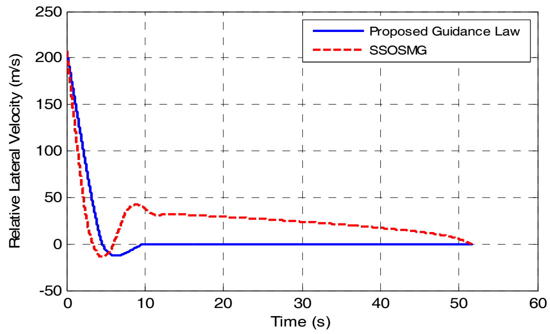

Fig. 13. Relative Lateral Velocity (V_q) in the Second Scenario

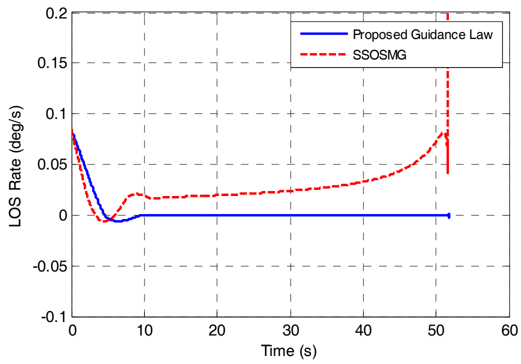

Fig. 14. Line of Sight rate in the Second Scenario

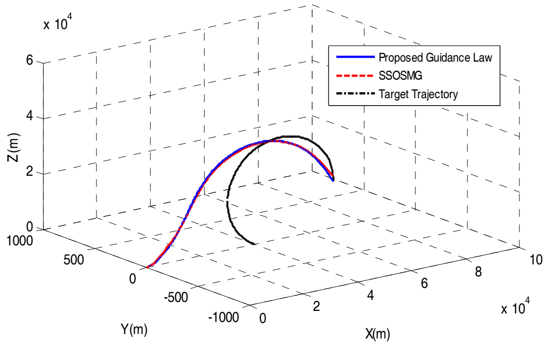

Fig. 15. Interception Trajectory in the second Scenario

Conclusion

In this paper, a new adaptive smooth second-order sliding mode guidance law is proposed. The finite time stabilization of the closed-loop system is proved using Lyapunov method. This guidance law guarantees finite time convergence of the relative lateral velocity as sliding variable in the presence of target acceleration as uncertainty and generates smooth acceleration commands as control signals. Simulation results show the effectiveness of the proposed guidance law in comparison with a homogeneity-based second-order sliding mode guidance law.

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