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Velocity and Body Rate Control of a Spacecraft Using Robust Passivity-Based Control

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This paper considers the problem of asymptotic stabilizing of velocity and body rates of a spacecraft in the presence of uncertainties and external disturbances. One of the important methods in controller design for nonlinear systems; is designing based on the passivity concept. This concept which provides a useful tool for analysis of nonlinear systems has been also used for asymptotic stabilizing of nonlinear dynamical systems especially mechanical systems. The passivity-based control law is a static output feedback and has valuable features. Because of existence of uncertainties and external disturbances in the state-space of equations of physical systems; first the robust version of passivity-based control method, which is recently developed in literature, is given and the control law for nonlinear uncertain systems with affine structure is presented. Then, this approach is used in controller design for a spacecraft. Since, this paper considers only the stabilization of velocity and body rates, therefore the reduced-order model is extracted from the state-space equation of a spacecraft with six degree of freedom and then the robust control law is designed. Computer simulations show the efficiency of the proposed controller in robust asymptotic stabilizing of the velocity and body rate vectors of the spacecraft in the presence of uncertainties and external disturbances.

Keyword: Passivity-based control, Robust stabilization, Spacecraft

Nomenclature

x	states vector of the system
u	input vector of the system
y	output vector of the system
r	position of the spacecraft
v	velocity of the spacecraft
q	quaternion of the spacecraft
ω	body rate of the spacecraft
m	mass of the spacecraft
ρ	the distances from the center of mass of the points where the forces are applied
F	force vector
T	torque vector
J	moments of inertia

Introduction

Controller design for spacecraft is an important problem which has been studied in literatures. For instance in [1], a controller is designed based on the sliding mode method. Authors of [2]; proposed a controller based on state dependent Riccati equation. Controller design for spacecraft stabilization based on backstepping adaptive sliding mode was given in [3].
Practically; in the state space equations of a spacecraft system, like other dynamical systems; may be uncertainty due to, external disturbances, parameter uncertainties or unknown nonlinear function which may be caused by inaccurate modeling or model reduction. Therefore, the proposed controllers should have a robust manner in the presence of uncertainties.

In [4]; a robust tracking control under input saturation was designed. Also, finite-time controllerfor robust stabilization of spacecraft were presented in [5,6].

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Among the nonlinear control method; passivity-based control is an important category [7-11]. Passivity-based control has application in many engineering problems like electromagnetic systems, process control, motor control, power electronics, mechanical systems [11-15].
There is a valuable feature for passive systems which can be stabilized utilizing a static output feedback.
If a nonlinear passive system be zero-state observable then will be asymptotically stabilized by a static output feedback like $u = -\phi(y)$ (where $\phi(0) = 0$ and $y^T \phi(y) > 0$, for all $y \neq 0$).
In this regard, the Kalman-Yakubovich-Popov (KYP) lemma is an important tool .The robust version of this lemma is also proposed [16]. According to it, the robust version of passivity-based control is recently presented [16-20].
The goal of this paper is designing a controller for robust stabilizing the velocity and body rate vectors of a spacecraft in the presence of uncertainties and external disturbances. To achieve this purpose, the robust passivity-based control method is used. Computer simulations show the efficiency of the proposed controller in robust asymptotic stabilizing of the velocity and body rate vectors of the spacecraft.

Robust Passivity-Based (RPB) Control Method

In this section, the robust control law is proposed for nonlinear affine systems in the presence of uncertainties and external disturbances, based on RPB control method. The effect of these unknown terms can be removed by a RPB controller which also asymptotically stabilizes the nonlinear system. Consider the following uncertain nonlinear system:

$$\begin{cases} \dot{x} = f(x) + G(x)(u + D(x) + \Delta(x)) \\ y = h(x) \end{cases} \tag{1}$$

where $x \in R^n$ represents the state vector, $u \in R^m$ is the input vector and $y \in R^m$ is the output vector. Also, $f(x)$ and $G(x)$ are Locally Lipschitz functions and $h(x)$ is a continuous vector function (where $f(0) = h(0) = 0$). $D(x)$ is the external disturbance and $\Delta(x)$ is the unknown uncertainties.
First, suppose $D(x) = \Delta(x) = 0$, therefore the nominal system is as follows:

$$\begin{cases} \dot{x} = f(x) + G(x)u \\ y = h(x) \end{cases} \tag{2}$$

System (2) is passive, if there exist a positive semi-definite function $S: x \to R$ (which $S(0) = 0$) such that [9]:

$$\dot{S} \leq y^T u \tag{3}$$

KYP Lemma 1 [9]**:** Consider system (2). Suppose that exist a positive semi-definite function $S: x \to R$, with $S(0) = 0$ for system (2), such that:

$$\begin{cases} \dfrac{\partial S}{\partial x} f(x) \leq 0 \\ \dfrac{\partial S}{\partial x} G(x) = h^T(x) \end{cases} \tag{4}$$

then the system (2) is passive.

Definition 1 [9]: Consider system (2), if for $y \equiv 0$ and $u \equiv 0$, there is not any solution for $\dot{x} = f(x, 0)$, except $x(t) \equiv 0$ then system (2) is zero-state observable.

Theorem 1:Suppose that system (2) is passive with a radially unbounded positive definite storage function $S(x)$ and it is zero-state observable, then its equilibrium point $x = 0$, can be globally asymptotically stabilized by the following control law:

$$u = -\phi(y) \tag{5}$$

where $\phi(.)$ is a function ($\phi(0) = 0$) that is Locally Lipschitz, and for all $y \neq 0$, $y^T \phi(y) > 0$.
Proof: [9].

Now consider the system (1) and suppose $D(x)$ and $\Delta(x)$ are non-zero. The following theorem which is based on the robust version of KYP lemma, gives the sufficient conditions for designing a control law.

Theorem 2: Consider the zero-state observable nonlinear system (1), and suppose there exists a positive definite storage function $S(x)$ such that:

$$\frac{\partial S}{\partial x} f(x) \leq 0 \tag{6}$$

$$\frac{\partial S}{\partial x} G(x) = h^T(x) \tag{7}$$

and

$$\|D(x)\| \leq \lambda_1 \|y\| \varphi_1^2(x), \ \lambda_1 > 0 \tag{8}$$

$$\|\Delta(x)\| \leq \lambda_2 \|y\| \varphi_2^2(x), \ \lambda_2 > 0 \tag{9}$$

where λ_1, λ_2 are known positive constants and $\varphi_1(x)$, $\varphi_2(x)$ are known functions. Then the following robust control law asymptotically stabilizes the nonlinear system (1) in the presence of uncertainties and external disturbances.

$$u = -k_1(y \varphi_1^2(x) + y \varphi_2^2(x)) - \phi(y) \tag{10}$$

where k_1 is a positive constant which is $k_1 > \max\{\lambda_1, \lambda_2\}$, and $\phi(y)$ satisfies the inequality $y^T \phi(y) > 0$ for all $y \neq 0$.

Proof: [17].

Application of RPB Method for Spacecraft Stabilization

In this section the proposed method (RPB) is used for robust stabilizing of the velocity and body rate vectors of a spacecraft.

The equations of a spacecraft with six degrees of freedom are as follows [2]:

$$\begin{bmatrix}\dot{r}\\ \dot{v}\\ \dot{q}\\ \dot{\omega}\end{bmatrix} = \begin{bmatrix}-\tilde{\omega}r + v\\ -\tilde{\omega}v\\ (1/2)\Omega(\omega)q\\ -J^{-1}\tilde{\omega}J\omega\end{bmatrix} + \begin{bmatrix}0_{3\times3} & 0_{3\times3}\\ (1/m)I_{3\times3} & 0_{3\times3}\\ 0_{4\times3} & 0_{4\times3}\\ J^{-1}\bar{\rho} & J^{-1}\end{bmatrix}\begin{bmatrix}F\\ \mathrm{T}\end{bmatrix} \quad (11)$$

where $r \in R^3$, $v \in R^3$, $q \in R^4$ and $\omega \in R^3$ are position, velocity of the spacecraft, quaternion and body rates of the spacecraft, respectively. The mass of the spacecraft is *m*. The distances from the center of mass of the points where the forces are applied are $\rho \in R^3$, and $\bar{\rho}$ represents the elements of the skew symmetric matrix ρ. Also, $F \in R^3$ includes the elements of control force vector and $T \in R^3$ is a vector of control torque of the spacecraft. The moments of inertia are $J \in R^{3\times3}$.Furthermore, of $\tilde{\omega}$ and $\Omega(\omega)$ are: where $r \in R^3$, $v \in R^3$, $q \in R^4$ and $\omega \in R^3$ are position, velocity of the spacecraft, quaternion and body rates of the spacecraft, respectively. The mass of the spacecraft is *m*. The distances from the center of mass of the points where the forces are applied are $\rho \in R^3$, and $\bar{\rho}$ represents the elements of the skew symmetric matrix ρ. Also, $F \in R^3$ includes the elements of control force vector and $T \in R^3$ is a vector of control torque of the spacecraft. The moments of inertia are $J \in R^{3\times3}$ Furthermore, of $\tilde{\omega}$ and $\Omega(\omega)$ are:

$$\tilde{\omega} = \begin{bmatrix}0 & -\omega_3 & \omega_2\\ \omega_3 & 0 & -\omega_1\\ -\omega_2 & \omega_1 & 0\end{bmatrix}, \Omega(\omega) = \begin{bmatrix}0 & -\omega_1 & -\omega_2 & -\omega_3\\ \omega_1 & 0 & \omega_3 & -\omega_2\\ \omega_2 & -\omega_3 & 0 & \omega_1\\ \omega_3 & \omega_2 & -\omega_1 & 0\end{bmatrix}$$

Here, the purpose is to asymptotically stabilize the velocity and body rates of the spacecraft. Since, in dynamical equations of $\dot{v}$ and $\dot{\omega}$ (refer to equations (11)), the state variables r and q ,have not been appeared; thus in order to stabilize v and ω the following reduced-order equations may be considered.

$$\begin{bmatrix}\dot{v}\\ \dot{\omega}\end{bmatrix} = \begin{bmatrix}-\tilde{\omega}v\\ -J^{-1}\tilde{\omega}J\omega\end{bmatrix} + \begin{bmatrix}(1/m)I_{3\times3} & 0_{3\times3}\\ 0_{3\times3} & J^{-1}\end{bmatrix}\begin{bmatrix}F\\ T\end{bmatrix} \quad (12)$$

Considering the state-space equations(12) in the structure of equation (2), then:

$$\dot{x} = \begin{bmatrix}\dot{v}\\ \dot{\omega}\end{bmatrix}, f = \begin{bmatrix}-\tilde{\omega}v\\ -J^{-1}\tilde{\omega}J\omega\end{bmatrix}, G = \begin{bmatrix}(1/m)I_{3\times3} & 0_{3\times3}\\ 0_{3\times3} & J^{-1}\end{bmatrix},$$

$$u = \begin{bmatrix}F\\ T\end{bmatrix} \quad (13)$$

Also, assume that the $D(v,\omega)$ and $\Delta(v,\omega)$ terms which may cause by inaccurate modeling or model reduction, are exist.

$$\begin{bmatrix}\dot{v}\\ \dot{\omega}\end{bmatrix} = \begin{bmatrix}-\tilde{\omega}v\\ -J^{-1}\tilde{\omega}J\omega\end{bmatrix} + \begin{bmatrix}(1/m)I_{3\times3} & 0_{3\times3}\\ 0_{3\times3} & J^{-1}\end{bmatrix} \times\left(\begin{bmatrix}F\\ T\end{bmatrix} + D(v,\omega) + \Delta(v,\omega)\right) \quad (14)$$

If $S(v,\omega) = (1/2)v^T v + (1/2)\omega^T \omega$ be assumed as a candidate of the storage function for the system (14), then:

$$\begin{aligned}\frac{\partial S}{\partial x} f(x) &= (1/2)\dot{v}^T v + (1/2)v^T \dot{v}\\ &+ (1/2)\dot{\omega}^T \omega + (1/2)\omega^T \dot{\omega}\\ &= -(1/2)\omega^T\left[(J^{-1}\tilde{\omega}J)^T + (J^{-1}\tilde{\omega}J)\right]\omega\end{aligned} \quad (15)$$

with numerical analysis, it can be shown that the equation (15) is negative definite therefore the condition (6) is satisfied.

Now, according to condition (7), by substituting G from (13), then:

$$y^T = \frac{\partial S}{\partial x} G(x) = \begin{bmatrix}\frac{\partial S}{\partial v} & \frac{\partial S}{\partial w}\end{bmatrix}.\begin{bmatrix}(1/m)I_{3\times3} & 0_{3\times3}\\ 0_{3\times3} & J^{-1}\end{bmatrix} \quad (16)$$

therefore, if the output of (14) is defined as follow, the condition (7) will be also satisfied.

$$y = \begin{bmatrix}(1/m)v\\ (J^{-1})^T \omega\end{bmatrix} \quad (17)$$

Suppose that $D(x)$ and $\Delta(x)$ satisfying the condition (8) and (9). Now, the task is to design the control vector $u = \begin{bmatrix}F & T\end{bmatrix}^T$, according to the Theorem 2, for robust asymptotically stabilizing of the state vector $x = \begin{bmatrix}v & \omega\end{bmatrix}^T$ in the presence of the non-zero terms $D(x)$ and $\Delta(x)$.

Computer Simulations

In this section the proposed controller is utilized for the system (14) and the time-response of the elements of state vectors (i.e. v_i and ω_i : for $i=1, 2, 3$) and also the control inputs (i.e. F_i and T_i : $i=1, 2, 3$) are presented.

In computer simulations the following vectors and parameters are used.

$$J=\begin{bmatrix} 3000 & -300 & -500 \\ -300 & 3000 & -400 \\ -500 & -400 & 3000 \end{bmatrix} kg\, m^2$$

$$m = 3000\, kg$$

$$D(v,\omega)=\begin{bmatrix} -0.3 & -0.2 & -0.1 & +0.2 & -0.1 & -0.2 \\ -0.2 & -0.1 & -0.2 & -0.2 & -0.3 & -0.2 \\ +0.2 & -0.1 & -0.2 & +0.1 & +0.3 & -0.1 \\ +0.3 & -0.5 & -0.3 & +0.4 & -0.3 & +0.2 \\ +0.1 & -0.2 & -0.1 & +0.2 & +0.1 & +0.1 \\ +0.2 & -0.1 & -0.2 & -0.5 & +0.3 & +0.1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

$$\Delta(v,\omega)=\begin{bmatrix} -0.2 & -0.2 & -0.3 & +0.2 & -0.1 & -0.2 \\ -0.4 & -0.1 & +0.3 & +0.2 & -0.3 & -0.2 \\ +0.2 & -0.1 & -0.2 & +0.1 & +0.3 & -0.1 \\ -0.2 & -0.5 & -0.3 & -0.6 & -0.3 & -0.2 \\ +0.2 & -0.4 & -0.1 & +0.2 & -0.2 & +0.1 \\ +0.1 & -0.1 & -0.2 & +0.5 & -0.1 & -0.1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

From equation (17):

$$y=\begin{bmatrix} (1/m)v \\ (J^{-1})^T\,\omega \end{bmatrix}=\begin{bmatrix} (1/m)I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & (J^{-1})^T \end{bmatrix}x \tag{18}$$

Since

$$\left\|\begin{bmatrix} (1/m)I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & (J^{-1})^T \end{bmatrix}\right\|=1$$

Therefore,

$$\|y\|\le\|x\| \tag{19}$$

$$\|D\|\le 0.9332\|x\| \tag{20}$$

$$\|\Delta\|\le 0.9856\|x\| \tag{21}$$

Now, by choosing $\lambda_1=\lambda_2=5$ and $\varphi_1(x)=\varphi_2(x)=1$ therefore, conditions (8) and (9) are satisfied.

Finally from (10), the feedback control law can be design as follows:

$$u=-k_1(y+y)-\phi(y) \tag{22}$$

where $k_1=2000$, and $\phi(y)=2y$ is chosen.

Figures 1 and 2 show the robust asymptotically stabilization of the velocity and body rate vectors, respectively.

Also, Figures 3 and 4 represent the time history of control force and control torque vectors, respectively. According to the simulation results, it is obvious that the proposed method guarantees the robust asymptotically stabilization of the velocity and body rates of the spacecraft.

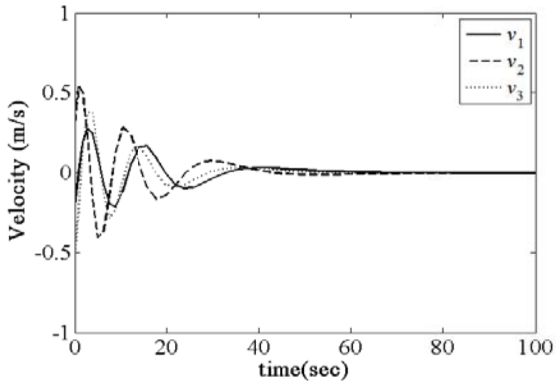

Fig. 1. Trajectory of state variables v_1, v_2, v_3

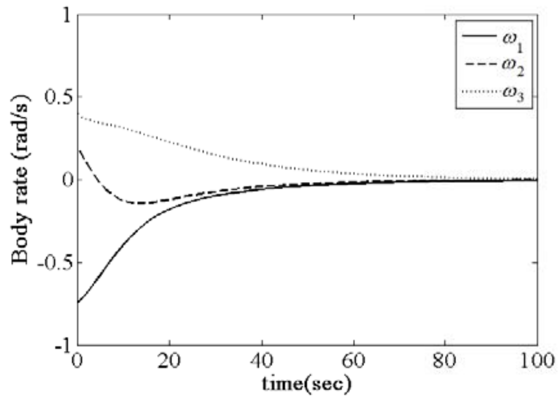

Fig. 2. Trajectory of state variables $\omega_1, \omega_2, \omega_3$

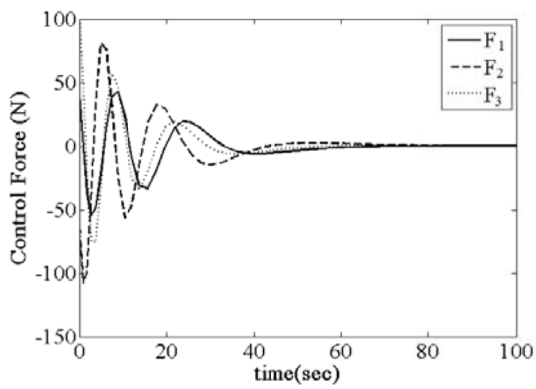

Fig. 3. Control forces of spacecraft

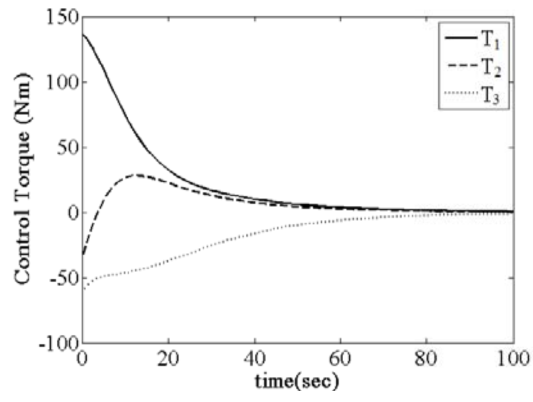

Fig. 4. Control torques of spacecraft

The initial conditions of the velocity and body rates are considered as follows:

$$v(0)=\begin{bmatrix}-0.2 & 0.3 & -0.5\end{bmatrix}^T$$

$$\omega(0)=\begin{bmatrix}-0.75 & 0.2 & 0.4\end{bmatrix}^T$$

Also, Figures 5-8 represent the time history of the velocity and body rates, control force and control torque for the following initial conditions:

$$v(0)=\begin{bmatrix}10 & -10 & 5\end{bmatrix}^T,\ \omega(0)=\begin{bmatrix}1 & 0.5 & -2\end{bmatrix}^T$$

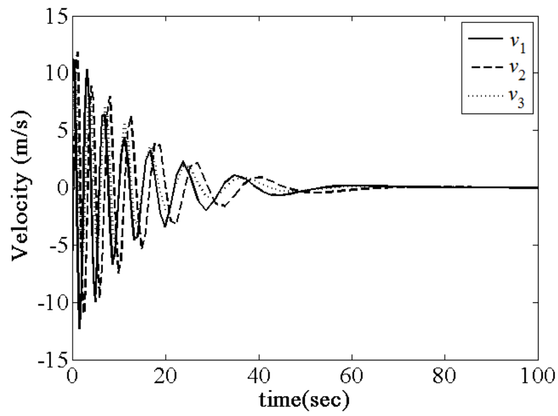

Fig. 5. Trajectory of state variables

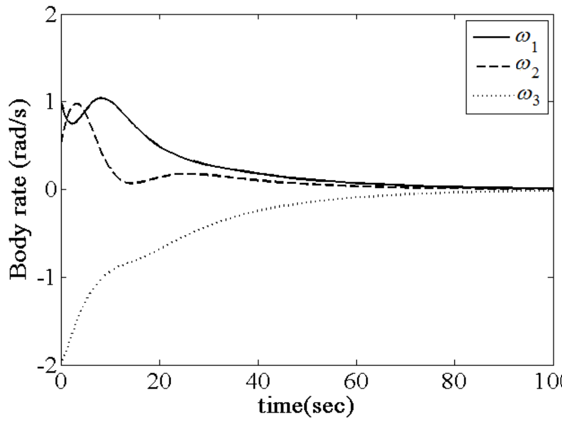

Fig. 6. Trajectory of state variables $\omega_1,\omega_2,\omega_3$

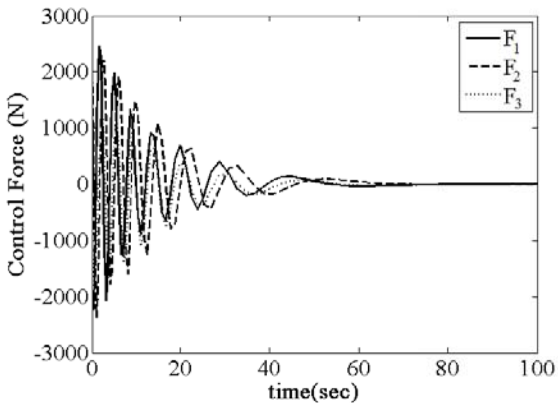

Fig. 7. Control forces of spacecraft

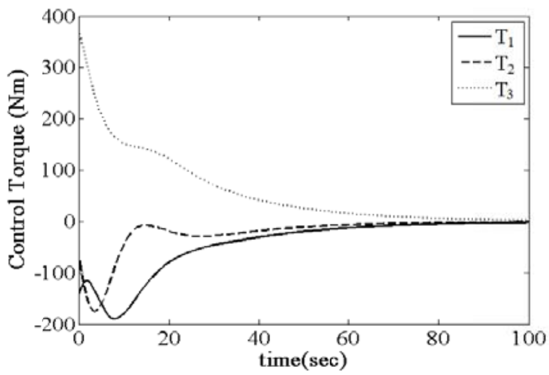

Fig. 8. Control torques of spacecraft

Conclusion

In this paper first the nonlinear passive systems was introduced and a robust stabilization control law was given based on the robust version of the KYP lemma. Then a robust passivity-based controller was designed for robust asymptotically stabilizing of the velocity and body rates of a spacecraft with six degrees of freedom. The computer simulations showed the efficiency of the proposed controller in terms of good characteristics of transient responses of the state variables and control inputs and also the robust asymptotic stabilizing of state variables.

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